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INTERNATIONAL UNION FOR THE PROTECTION OF NEW VARIETIES OF PLANTS

GENEVA

Associated Document
to the
General Introduction to the Examination
of Distinctness, Uniformity and Stability and the
Development of Harmonized Descriptions of New Varieties of Plants (document TG/1/3)

DOCUMENT TGP/10

"EXAMINING UNIFORMITY"

Section TGP/10.3.2: Recommended Statistical Methods: Offtypes

Document prepared by experts from the United Kingdom

to be considered by the

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SECTION 10.3.2
RECOMMENDED STATISTICAL METHODS: OFFTYPES

TESTING UNIFORMITY BY OFF-TYPES – FIXED POPULATION STANDARD

SUMMARY

1. This section describes the method of assessing uniformity by comparing the number of off-types observed to a fixed population standard. This is of particular use for self-pollinated and vegetatively propagated crops.

2. Uniformity of candidate varieties of self-pollinated and vegetatively propagated crops is normally assessed on the basis of the number of off-types recorded in tests. The maximum number of off-types that is acceptable should be chosen so that the probability of rejecting a candidate variety that should meet the crop standard is small. On the other hand the probability of accepting a candidate variety that has many more off-types than the standard of that crop should also be low.

3. The methods described here address the problem of choosing the maximum permitted number of off-types for different standards and sample sizes so that the probability of making errors is known and acceptable. The methods involve establishing a standard for the crop in question and then choosing the sample size and the number of off-types that best satisfy the risks that can be tolerated.

4. This document also outlines procedures for when more than a single test (more than one year for instance) is done and mentions the possibility of using sequential tests to minimize testing effort.

INTRODUCTION

5. When testing for uniformity on the basis of a sample, there will always be some risk of making a wrong decision. The risks can be reduced by increasing the sample size but at a greater cost. The aim of the statistical procedure described here is to achieve an acceptable balance between risks.

6. The procedures described below require the user to define an acceptance standard (called the *population standard*) for the crop in question. The methods described then show how to determine the sample size and the maximum number of off-types allowed for various levels of risks.

7. The population standard is the maximum percentage of off-types that would be accepted if all individuals of the variety could be examined.

UPOV RECOMMENDATIONS ON THE FIXED POPULATION STANDARD METHOD OF ASSESSING UNIFORMITY BY NUMBER OF OFF-TYPES

8. This method is recommended for use in assessing the uniformity by number of off-types in self-pollinated and vegetatively propagated crops.

9. The sample size and acceptable number of off-types employed depend on the crop. Recommended sample sizes and acceptable numbers of off-types for different crops are given in the Annex to TGP/10.3.

ERRORS IN TESTING FOR OFF-TYPES

10. As mentioned, there will be some risk of making wrong decisions. Two types of error exist:

(a) Declaring that the variety lacks uniformity when it in fact meets the standard for the crop. This is known as “type I error.”

(b) Declaring that the variety is uniform when it in fact does not meet the standard for the crop. This is known as “type II error.”

11. The types of error can be summarized in the following table:

True state of the variety	Decision made on variety	
	Acceptance as uniform	Rejection as non-uniform
uniform	correctly accepted	type I error
heterogeneous	type II error	correctly rejected

12. The probability of correctly accepting a uniform variety is called the acceptance probability and is linked to the probability of type I error by the relation:

“Acceptance probability” + “probability of type I error” = 100%

13. The probability of type II error depends on “how heterogeneous” the candidate variety is. If it is much more heterogeneous than the population standard then the probability of type II error will be small and we will have a small probability of accepting such a variety. If, on the other hand, the candidate variety is only slightly more heterogeneous than the standard, we will have a large probability of type II error. The probability of acceptance will approach the acceptance probability for a variety with a level of uniformity near to the population standard.

14. Because the probability of type II error is not fixed but depends on “how heterogeneous” the candidate variety is, this probability can be calculated for different degrees of heterogeneity. This document gives probabilities of type II error for three degrees of heterogeneity: 2, 5 and 10 times the population standard.

15. In general, the probability of making errors will be decreased by increasing the sample size and increased by decreasing the sample size.

16. For a given sample size, the balance between the probabilities of making type I and type II errors may be altered by changing the number of off-types allowed.

17. If the number of off-types allowed is increased, the probability of type I error is decreased but the probability of type II error is increased. On the other hand, if the number of off-types allowed is decreased, the probability of type I errors is increased while the probability of type II errors is decreased.

18. By allowing a very high number of off-types it will be possible to make the probability of type I errors very low (or almost zero). However, the probability of making type II errors will now become (unacceptably) high. If only a very low number of off-types are allowed, the result will be a small probability of type II errors and an (unacceptably) high probability of type I errors. This will be illustrated by examples.

EXAMPLES

Example 1

19. From experience, a reasonable standard for the crop in question is found to be 1%. So the population standard is 1%. Assume also that a single test with a maximum of 60 plants is done. From tables 4, 10 and 16 (chosen to give a range of target acceptance probabilities), the following schemes are found:

Scheme	Sample size	Target acceptance probability*	Maximum number of off-types
a	60	90%	2
b	53	90%	1
c	60	95%	2
d	60	99%	3

20. From the figures 4, 10 and 16, the following probabilities are obtained for the type I error and type II error for different percentages of off-types (denoted by P_2, P_5 and P_{10} for 2, 5 and 10 times the population standard).

* See paragraph 54

Scheme	Sample size	Maximum number of off-types	Probabilities of error (%)			
			Type I	Type II		
				P_2 = 2%	P_5 = 5%	P_{10} = 10%
a	60	2	2	88	42	5
b	53	1	10	71	25	3
c	60	2	2	88	42	5
d	60	3	0.3	97	65	14

21. The table lists four different schemes and they should be examined to see if one of them is appropriate to use. (Schemes a and c are identical since there is no scheme for a sample size of 60 with a probability of type I error between 5 and 10%). If it is decided to ensure that the probability of a type I error should be very small (scheme d) then the probability of the type II error becomes very large (97, 65 and 14%) for a variety with 2, 5 and 10% of off-types, respectively. The best balance between the probabilities of making the two types of error seems to be obtained by allowing one off-type in a sample of 53 plants (scheme b).

<u>Example 2</u>

22. In this example, a crop is considered where the population standard is set to 2% and the number of plants available for examination is only 6.

23. Using the tables and the figures 3, 9 and 15, the following schemes a-d are found:

Sche-me	Sample size	Acceptance probability	Maximum number of off-types	Probability of error (%)			
				Type I	Type II		
					P_2 = 4%	P_5 = 10%	P_{10} = 20%
a	6	90	1	0.6	98	89	66
b	5	90	0	10	82	59	33
c	6	95	1	0.6	98	89	66
d	6	99	1	0.6	98	89	66
e	6		0	11	78	53	26

24. Scheme e of the table is found by applying the formulas (1) and (2) shown later in this document.

25. This example illustrates the difficulties encountered when the sample size is very low. The probability of erroneously accepting a heterogeneous variety (a type II error) is large for all the possible situations. Even when all five plants must be uniform for a variety to be accepted (scheme b), the probability of accepting a variety with 20% of off-types is still 33%.

26. It should be noted that a scheme where all six plants must be uniform (scheme e) gives slightly smaller probabilities of type II errors, but now the probability of the type I error has increased to 11%.

27. However, scheme e may be considered the best option when only six plants are available in a single test for a crop where the population standard has been set to 2%.

Example 3

28. In this example we reconsider the situation in example 1 but assume that data are available for two years. So the population standard is 1% and the sample size is 120 plants (60 plants in each of two years).

29. The following schemes and probabilities are obtained from the tables and figures 4, 10 and 16:

Scheme	Sample size	Acceptance probability	Maximum number of off-types	Probability of error (%)			
				Type I	Type II		
					$P_2 = 2\%$	$P_5 = 5\%$	$P_{10} = 10\%$
a	120	90	3	3	78	15	<0.1
b	110	90	2	10	62	8	<0.1
c	120	95	3	3	78	15	<0.1
d	120	99	4	0.7	91	28	1

30. Here the best balance between the probabilities of making the two types of error is obtained by scheme c, i.e. to accept after two years a total of three off-types among the 120 plants examined.

31. Alternatively a two-stage testing procedure may be set up. Such a procedure can be found for this case by using formulae (3) and (4) later in this document.

32. The following schemes can be obtained:

Scheme	Sample size	Acceptance probability	Largest number for acceptance after year 1	Largest number before reject in year 1	Largest number to accept after 2 years
e	60	90	can never accept	2	3
f	60	95	can never accept	2	3
g	60	99	can never accept	3	4
h	58	90	1	2	2

33. Using the formulas (3), (4) and (5) the following probabilities of errors are obtained:

Scheme	Probability of error (%)				Probability of testing in a second year
	Type I	Type II			
		P_2 = 2%	P_5 = 5%	P_{10} = 10%	
e	4	75	13	0.1	100
f	4	75	13	0.1	100
g	1	90	27	0.5	100
h	10	62	9	0.3	36

34. Schemes e and f (which are identical) result in a probability of 4% for rejecting a uniform variety (type I error) and a probability of 13% for accepting a variety with 5% off-types (type II error). The decision is:

- Never accept the variety after 1 year
- More than 2 off-types in year 1: reject the variety and stop testing
- Between and including 0 and 2 off types in year 1: do a second year test
- At most 3 off-types after 2 years: accept the variety
- More than 3 off-types after 2 years: reject the variety

35. Alternatively, scheme h may be chosen but scheme g seems to have a too large probability of type II errors compared with the probability of type I error.

36. Scheme h has the advantage of often allowing a final decision to be taken after the first test (year) but, as a consequence, there is a higher probability of a type I error.

Example 4

37. In this example, we assume that the population standard is 3% and that we have 8 plants available in each of two years.

38. From the tables and figures 2, 8 and 14, we have:

Scheme	Sample size	Acceptance probability	Maximum number of off-types	Probability of error (%)			
				Type I	Type II		
					P_2 = 6%	P_5 = 15%	P_{10} = 30%
a	16	90	1	8	78	28	3
b	16	95	2	1	93	56	10
c	16	99	3	0.1	99	79	25

39. Here the best balance between the probabilities of making the two types of error is obtained by scheme a.

INTRODUCTION TO THE TABLES AND FIGURES

40. In the TABLES AND FIGURES section, there are 21 table and figure pairs corresponding to different combinations of population standard and acceptance probability. These are design to be applied to a single off-type test. An overview of the tables and the figures are given in table A.

41. Each table shows the maximum numbers of off-types (k) with the corresponding ranges in sample sizes (n) for the given population standard and acceptance probability. For example, in table 1 (population standard 5%, acceptance probability ≥ 90%), for a maximum set at 2 off-types, the corresponding sample size (n) is in the range from 11 to 22. Likewise, if the maximum number of off-types (k) is 10, the corresponding sample size (n) to be used should be in the range 126 to 141.

42. For small sample sizes, the same information is shown graphically in the corresponding figures (figures (1 to 21). These show the actual risk of rejecting a uniform variety and the probability of accepting a variety with a true proportion of off-types 2 times (2P), 5 times (5P) and 10 times (10P) greater than the population standard. (To ease the reading of the figure, lines connect the risks for the individual sample sizes, although the probability can only be calculated for each individual sample size).

Table A. Overview of table and figure 1 to 18.

Population standard %	Acceptance probability %	See table and figure no.
10	>90	19
10	>95	20
10	>99	21
5	>90	1
5	>95	7
5	>99	13
3	>90	2
3	>95	8
3	>99	14
2	>90	3
2	>95	9
2	>99	15
1	>90	4
1	>95	10
1	>99	16
0.5	>90	5
0.5	>95	11
0.5	>99	17
0.1	>90	6
0.1	>95	12
0.1	>99	18

43. When using the tables the following procedure is suggested:

(a) Choose the relevant population standard.

(b) Write down the different relevant decision schemes (combinations of sample size and maximum number of off-types), with the probabilities of type I and type II errors read from the figures.

(c) Choose the decision scheme with the best balance between the probabilities of errors.

44. The use of the tables and figures is illustrated in the example section.

DETAILED DESCRIPTION OF THE METHOD FOR ONE SINGLE TEST

45. The mathematical calculations are based on the binomial distribution and it is common to use the following terms:

(a) The percentage of off-types to be accepted in a particular case is called the “population standard” and symbolized by the letter P.

(b) The “acceptance probability” is the probability of accepting a variety with P% of off-types. However, because the number of off-types is discrete, the actual probability of accepting a uniform variety varies with sample size but will always be greater than or equal to the “acceptance probability.” The acceptance probability is usually denoted by 100 - α, where α is the percent probability of rejecting a variety with P% of off-types (i.e. type I error probability). In practice, many varieties will have less than P% off-types and hence the type I error will in fact be less than α for such varieties.

(c) The number of plants examined in a random sample is called the sample size and denoted by n.

(d) The maximum number of off-types tolerated in a random sample of size n is denoted by k.

(e) The probability of accepting a variety with more than P% off-types, say P_q% of off-types, is denoted by the letter β or by β_q.

(f) The mathematical formulae for calculating the probabilities are:

$$\alpha = 100 - 100 \sum_{i=0}^{k} \binom{n}{i} P^{i} (1 - P)^{n-i} \qquad (1)$$

$$\beta_q = 100 \sum_{i=0}^{k} \binom{n}{i} P_q^{i} (1 - P_q)^{n-i} \qquad (2)$$

P and P_q are expressed here as proportions, i.e. percents divided by 100.

MORE THAN ONE SINGLE TEST (YEAR)

46. Often a candidate variety is grown in two (or three years). The question then arises of how to combine the uniformity information from the individual years. Two methods will be described:

(a) Make the decision after two (or three) years based on the total number of plants examined and the total number of off-types recorded. (A combined test).

(b) Use the result of the first year to see if the data suggests a clear decision (reject or accept). If the decision is not clear then proceed with the second year and decide after the second year. (A two-stage test).

47. However, there are some alternatives (e.g. a decision may be made in each year and a final decision may be reached by rejecting the candidate variety if it shows too many off-types in both (or two out of three years)). Also there are complications when more than one single year test is done. It is therefore suggested that a statistician should be consulted when two (or more) year tests have to be used.

DETAILED DESCRIPTION OF THE METHODS FOR MORE THAN ONE SINGLE TEST

<u>Combined Test</u>

48. The sample size in test i is n_i. So after the last test we have the total sample size $n = \Sigma n_i$. A decision scheme is set in exactly the same way as if this total sample size had been obtained in a single test. Thus, the total number of off-types recorded through the tests is compared with the maximum number of off-types allowed by the chosen decision scheme.

<u>Two-stage Test</u>

49. The method for a two-year test may be described as follows: In the first year take a sample of size n. Reject the candidate variety if more than r_1 off-types are recorded and accept the candidate variety if less than a_1 off-types are recorded. Otherwise, proceed to the second year and take a sample of size n (as in the first year) and reject the candidate variety if the total number of off-types recorded in the two years' test is greater than r. Otherwise, accept the candidate variety. The final risks and the expected sample size in such a procedure may be calculated as follows:

$$\alpha = P(K_1 > r_1) + P(K_1 + K_2 > r \mid K_1)$$
$$= P(K_1 > r_1) + P(K_2 > r\text{-}K_1 \mid K_1)$$

$$= \sum_{i=r_1+1}^{n} \binom{n}{i} P^i (1-P)^{n-i} + \sum_{i=\alpha_1}^{r_1} \binom{n}{i} P^i (1-P)^{n-i} \sum_{j=r-i+1}^{n} \binom{n}{i} P^j (1-P)^{n-j} \qquad (3)$$

$$\beta_q = P(K_1 < \alpha_1) + P(K_1 + K_2 \leq r \mid K_1)$$
$$= P(K_1 < \alpha_1) + P(K_2 \leq r\text{-}K_1 \mid K_1)$$

$$= \sum_{i=0}^{\alpha_1-1} \binom{n}{i} P_q^i (1-P_q)^{n-i} + \sum_{i=\alpha_1}^{r_1} \binom{n}{i} P_q^i (1-P_q)^{n-i} \sum_{j=0}^{r-i} \binom{n}{i} P_q^j (1-P_q)^{n-j} \qquad (4)$$

$$n_e = n \left(1 + \sum_{i=\alpha_1}^{r_1} \binom{n}{i} P^i (1-P)^{n-i} \right) \qquad (5)$$

where

P = population standard
α = probability of actual type I error for P
β_q = probability of actual type II error for q P
n_e = expected sample size
r_1, a_1 and r are decision-parameters
P_q = q times population standard = q P
K_1 and K_2 are the numbers of off-types found in years 1 and 2 respectively.

50. The decision parameters, a_1, r_1 and r, may be chosen according to the following criteria:

(a) α must be less than α_0, where α_0 is the maximum type I error, i.e. α_0 is 100 minus the required acceptance probability
(b) β_q (for q=5) should be as small as possible but not smaller than α_0
(c) if β_q (for q=5) < α_0 n_e should be as small as possible.

51. However, other strategies are available. No tables/figures are produced here as there may be several different decision schemes that satisfy a certain set of risks. It is suggested that a statistician should be consulted if a 2-stage test (or any other sequential tests) is required.

SEQUENTIAL TESTS

52. The two-stage test mentioned above is a type of sequential test where the result of the first stage determines whether the test needs to be continued for a second stage. Other types of sequential tests may also be applicable. It may be relevant to consider such tests when the practical work allows analyses of off-types to be carried out at certain stages of the examination. The decision schemes for such methods can be set up in many different ways and it is suggested that a statistician should be consulted when sequential methods are to be used.

NOTE ON TYPE I AND TYPE II ERRORS

53. Because the number of off-types is discrete, we cannot in general obtain type I-errors that are nice pre-selected figures. The scheme a of example 2 with 6 plants above showed that we could not obtain an α of 10% - our actual α became 0.6%. Increasing the sample size will result in varying α and β values. Figure 3 - as an example - shows that α gets closer to its nominal values at certain sample sizes and that this is also the sample size where β is relatively small. It is also seen that increasing the sample size for fixed acceptance probability is not always advantageous. For instance a sample size of five gives α = 10% and β_2 = 82% whereas a sample size of six gives α = 0.6% and β_2 = 98%. It appears that the sample sizes, which give α-values in close agreement with the acceptance probability are the largest in the range of sample sizes with a specified maximum number of off-types. Thus, the smallest sample sizes in the range of sample sizes with a given maximum number of off-types should be avoided.

DEFINITION OF STATISTICAL TERMS AND SYMBOLS

54. The statistical terms and symbols used have the following definitions:

Population standard. The percentage of off-types to be accepted if all the individuals of a variety could be examined. The population standard is fixed for the crop in question and is based on experience.

Acceptance probability. The probability of accepting a uniform variety with P% of off-types. Here P is population standard. However, note that the actual probability of accepting a uniform variety will always be greater than or equal to the acceptance probability in the heading of the table and figures. The probability of accepting a uniform variety and the probability of a type I error sum to 100%. For example, if the type I error probability is 4%, then the probability of accepting a uniform variety is 100 – 4 = 96%, see e.g. figure 1 for n=50). The type I error is indicated on the graph in the figures by the sawtooth peaks between 0 and the upper limit of type I error (for instance 10 on figure 1). The decision schemes are defined so that the actual probability of accepting a uniform variety is always greater than or equal to the acceptance probability in the heading of the table.

Type I error: The error of rejecting a uniform variety.

Type II error: The error of accepting a variety that is too heterogeneous.

P Population standard

P_q The assumed true percentage of off-types in a heterogeneous variety. $P_q = q\ P$.

In the present document q is equal to 2, 5 or 10. These are only 3 examples to help the visualization of type II errors. The actual percentage of off-types in a variety may take any value. For instance we may examine different varieties which in fact may have respectively 1.6%, 3.8%, 0.2%, … of off-types.

n Sample size
k Maximum number of off-types allowed
α Probability of type I error
β Probability of type II error

TABLES AND FIGURES

Table and figure 1: **Population Standard = 5%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

n	k
1- 2	0
3- 10	1
11 22	2
23- 35	3
36- 49	4
50- 63	5
64- 78	6
79- 94	7
95- 109	8
110- 125	9
126 141	10
142- 158	11
159- 174	12
175- 191	13
192- 207	14
208- 224	15
225- 241	16
242- 258	17
259- 275	18
276- 292	19
293- 310	20
311- 327	21
328- 344	22
345- 362	23
363- 379	24
380- 397	25
398- 414	26
415- 432	27
433- 449	28
450- 467	29
468- 485	30
486- 503	31
504- 520	32
521- 538	33
539- 556	34
557- 574	35
575- 592	36
593- 610	37
611- 628	38
629- 646	39
647- 664	40
665- 682	41
683- 700	42
701 718	43
719- 736	44
737- 754	45
755- 772	46
773 791	47
792- 809	48
810- 827	49
828- 845	50
846- 864	51
865- 882	52
883- 900	53
901- 918	54
919- 937	55
938- 955	56
956- 973	57
974- 992	58
993-1010	59

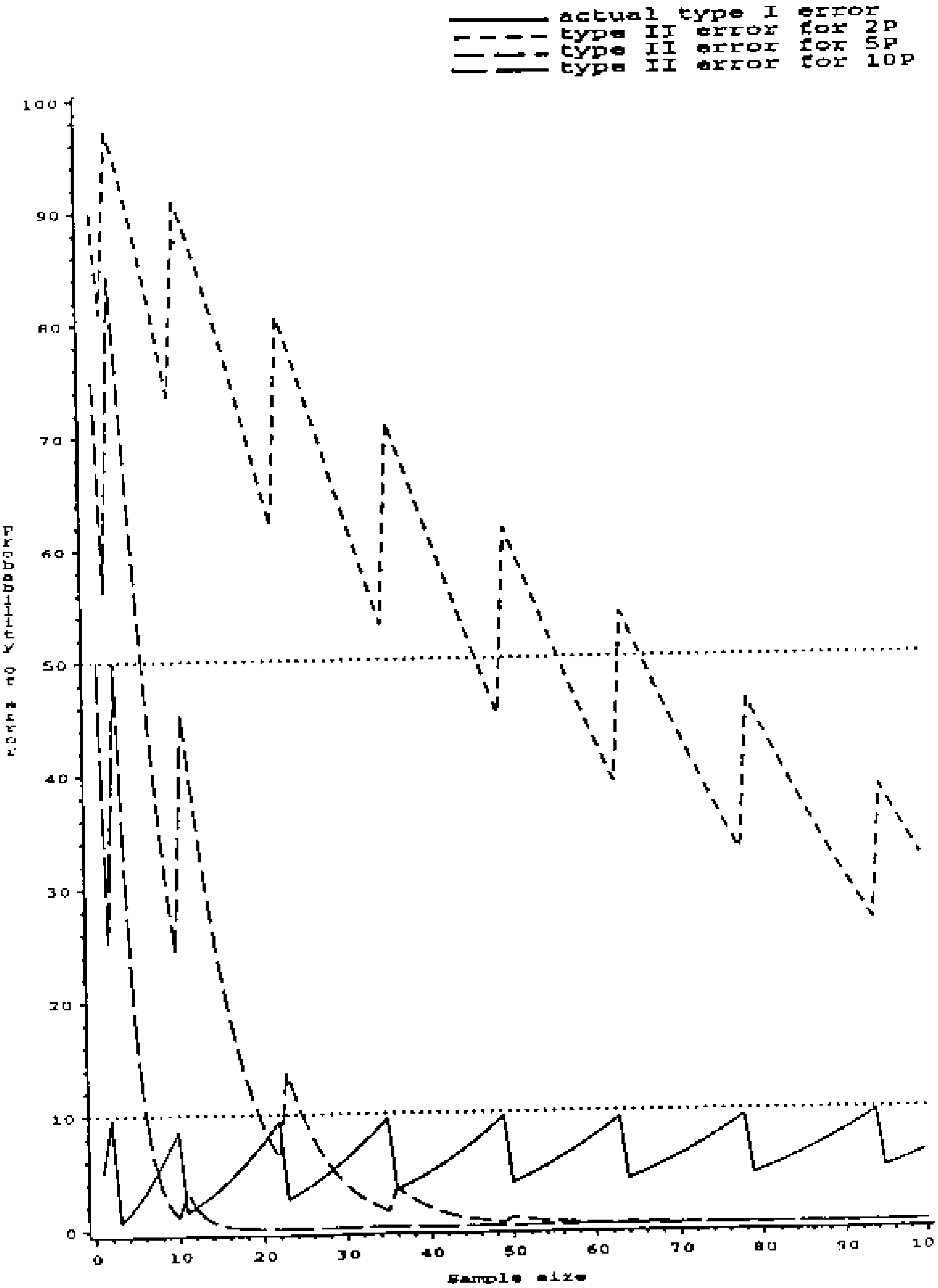


Table and figure 2: **Population Standard = 3%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

n	k
1- 3	0
4- 17	1
18- 37	2
38- 58	3
59- 81	4
82- 105	5
106- 130	6
131- 156	7
157- 182	8
183- 208	9
209- 235	10
236- 262	11
263- 289	12
290- 317	13
318- 345	14
346- 373	15
374- 401	16
402- 429	17
430- 457	18
458- 486	19
487- 515	20
516- 543	21
544- 572	22
573- 601	23
602- 630	24
631- 659	25
660- 689	26
690- 718	27
719- 747	28
748- 777	29
778- 806	30
807- 836	31
837- 865	32
866- 895	33
896- 925	34
926- 955	35
956- 984	36
985-1014	37
1015-1044	38
1045-1074	39
1075-1104	40
1105-1134	41
1135-1164	42
1165-1195	43
1196-1225	44
1226-1255	45
1256-1285	46
1286-1315	47
1316-1346	48
1347-1376	49
1377-1406	50
1407-1437	51
1438-1467	52
1468-1498	53
1499-1528	54

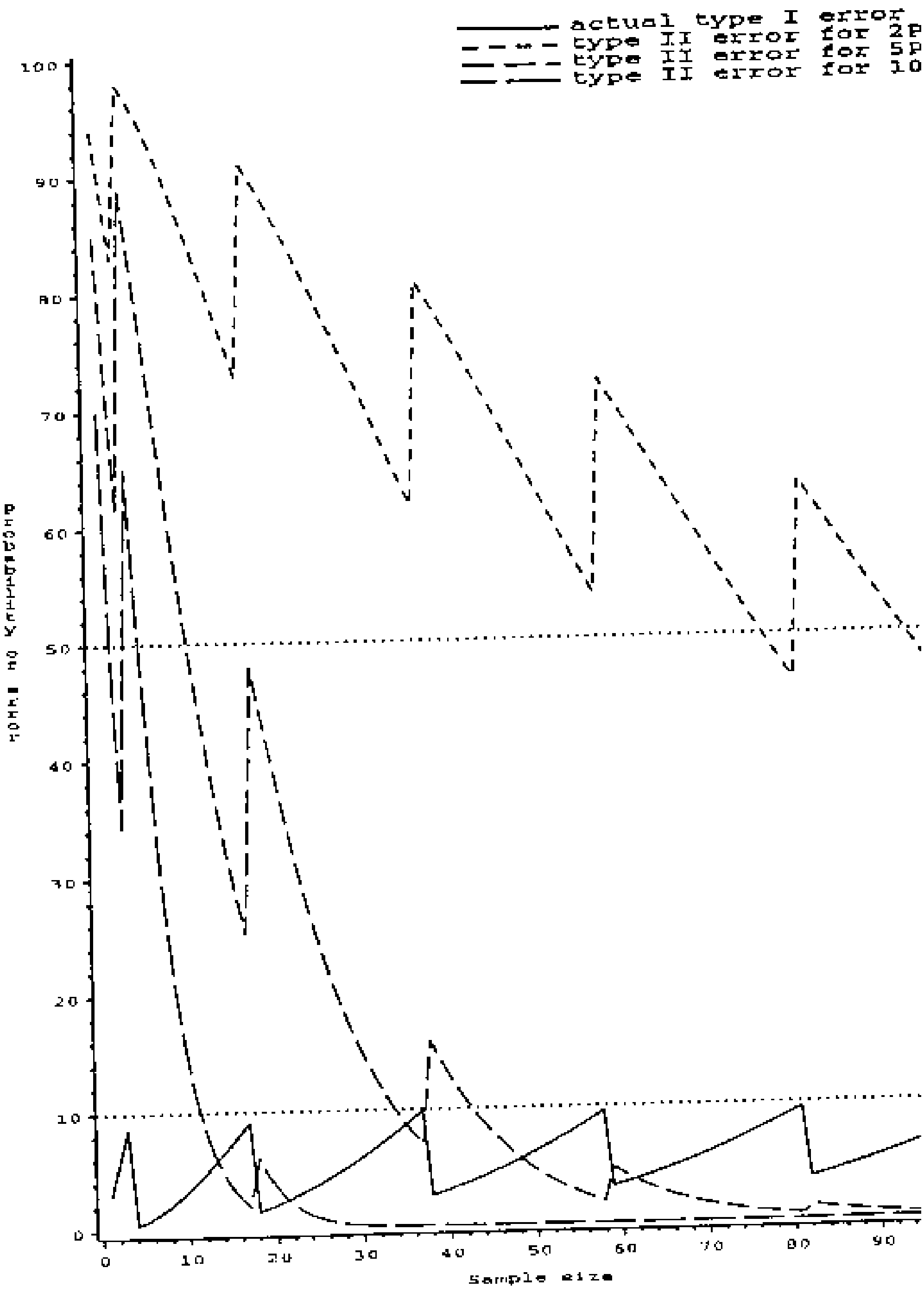


Table and figure 3: **Population Standard = 2%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

n	k
1- 5	0
6- 26	1
27- 55	2
56- 87	3
88- 122	4
123- 158	5
159- 195	6
196- 233	7
234- 272	8
273- 312	9
313- 352	10
353- 393	11
394- 433	12
434- 475	13
476- 516	14
517- 558	15
559- 600	16
601- 643	17
644- 685	18
686- 728	19
729- 771	20
772- 814	21
815- 857	22
858- 901	23
902- 944	24
945- 988	25
989-1032	26
1033-1076	27
1077-1120	28
1121-1164	29
1165-1208	30
1209-1252	31
1253-1297	32
1298-1341	33
1342-1386	34
1387-1431	35
1432-1475	36
1476-1520	37
1521-1565	38
1566-1610	39
1611-1655	40
1656-1700	41
1701-1745	42
1746-1790	43
1791-1835	44
1836-1881	45
1882-1926	46
1927-1971	47
1972-2000	48

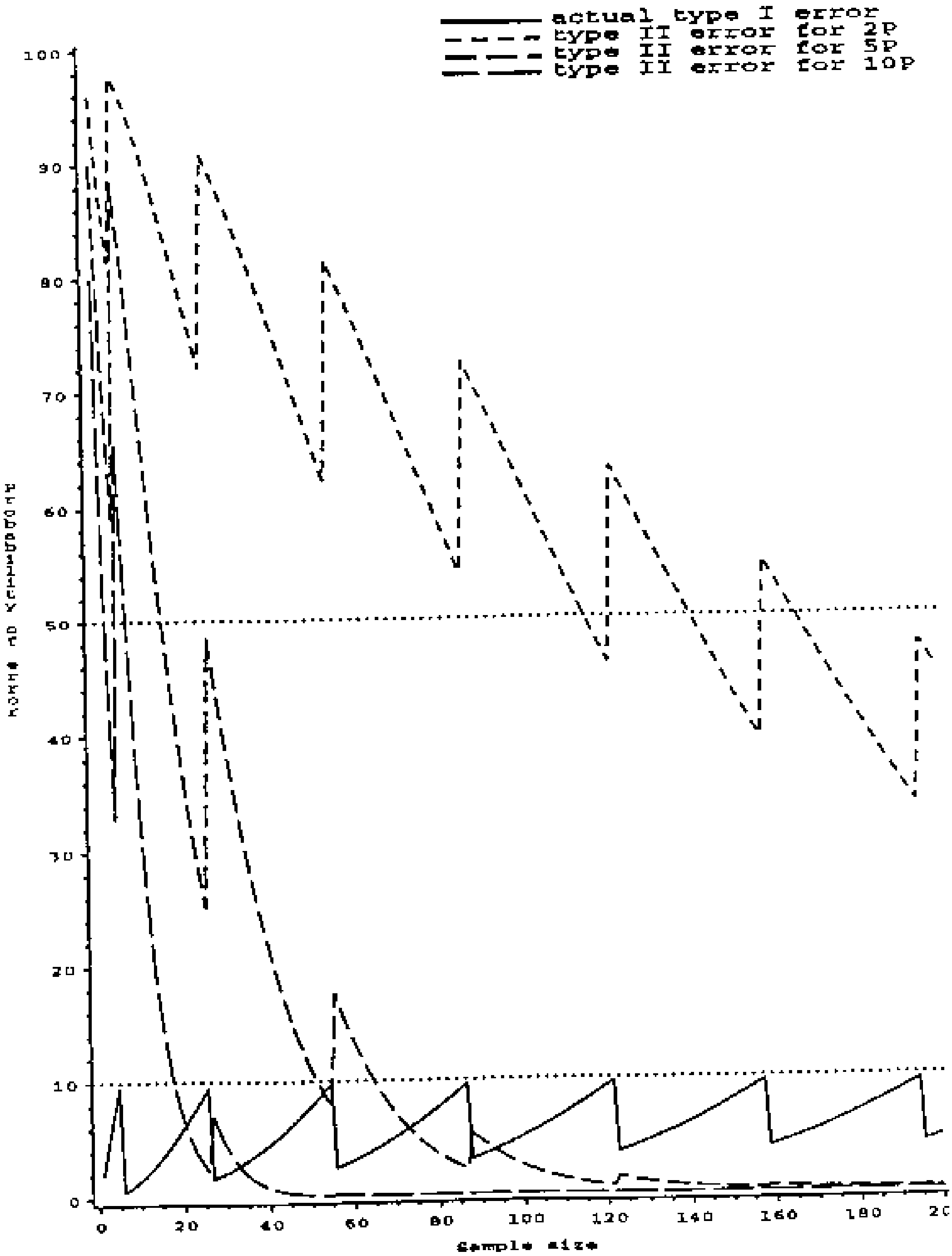


Table and figure 4: **Population Standard = 1%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

n	k
1- 10	0
11- 53	1
54- 110	2
111- 175	3
176- 244	4
245- 316	5
317- 390	6
391- 466	7
467- 544	8
545- 623	9
624- 703	10
704- 784	11
785- 866	12
867- 948	13
949-1031	14
1032-1115	15
1116-1199	16
1200-1284	17
1285-1369	18
1370-1454	19
1455-1540	20
1541-1626	21
1627-1713	22
1714-1799	23
1800-1887	24
1888-1974	25
1975-2061	26
2062-2149	27
2150-2237	28
2238-2325	29
2326-2414	30
2415-2502	31
2503-2591	32
2592-2680	33
2681-2769	34
2770-2858	35
2859-2948	36
2949-3000	37

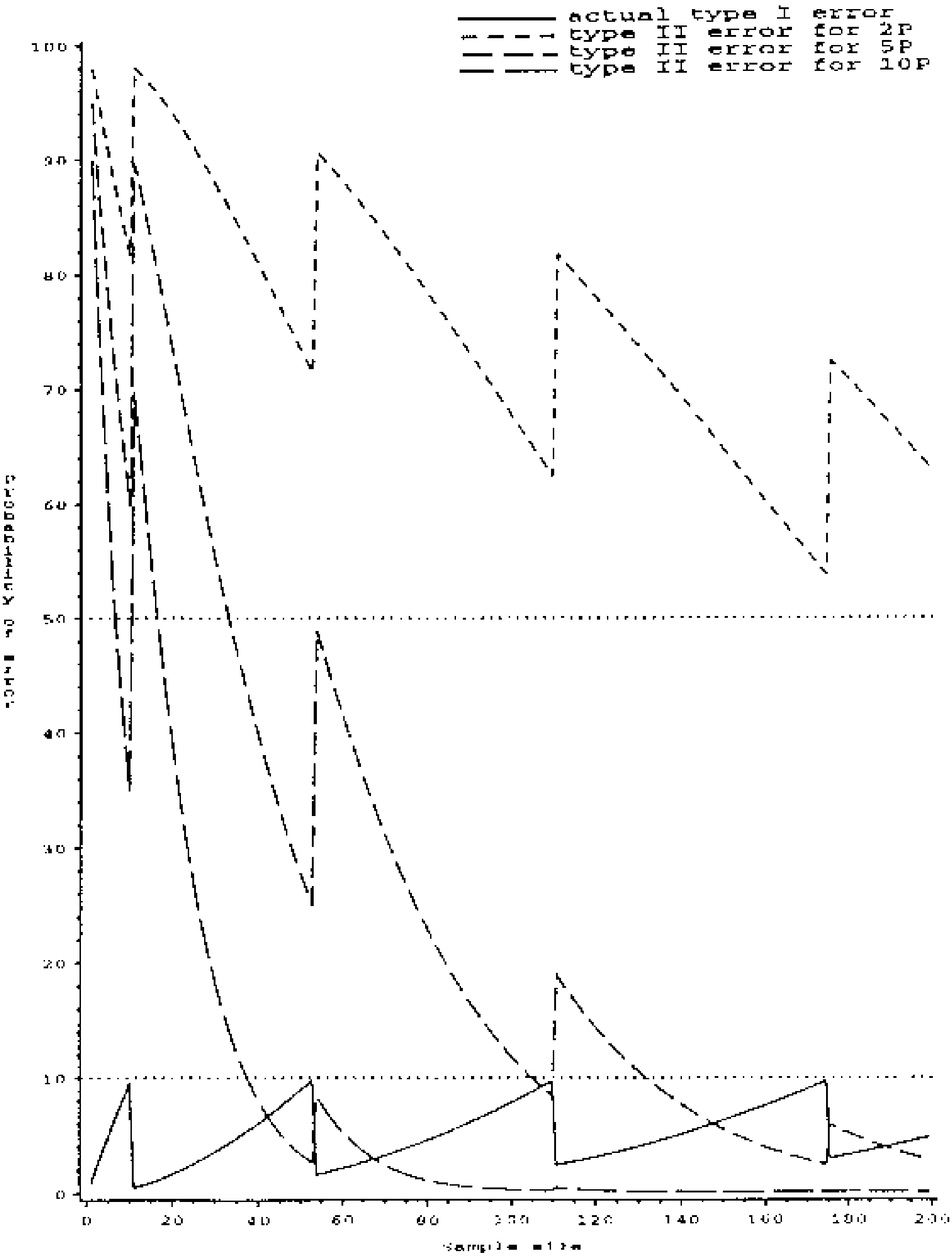


Table and figure 5: **Population Standard = .5%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

n		k
1-	21	0
22-	106	1
107-	220	2
221-	349	3
350-	487	4
488-	631	5
632-	780	6
781-	932	7
933-	1087	8
1088-	1245	9
1246-	1405	10
1406-	1567	11
1568-	1730	12
1731-	1895	13
1896-	2061	14
2062-	2228	15
2229-	2397	16
2398-	2566	17
2567-	2736	18
2737-	2907	19
2908-	3000	20

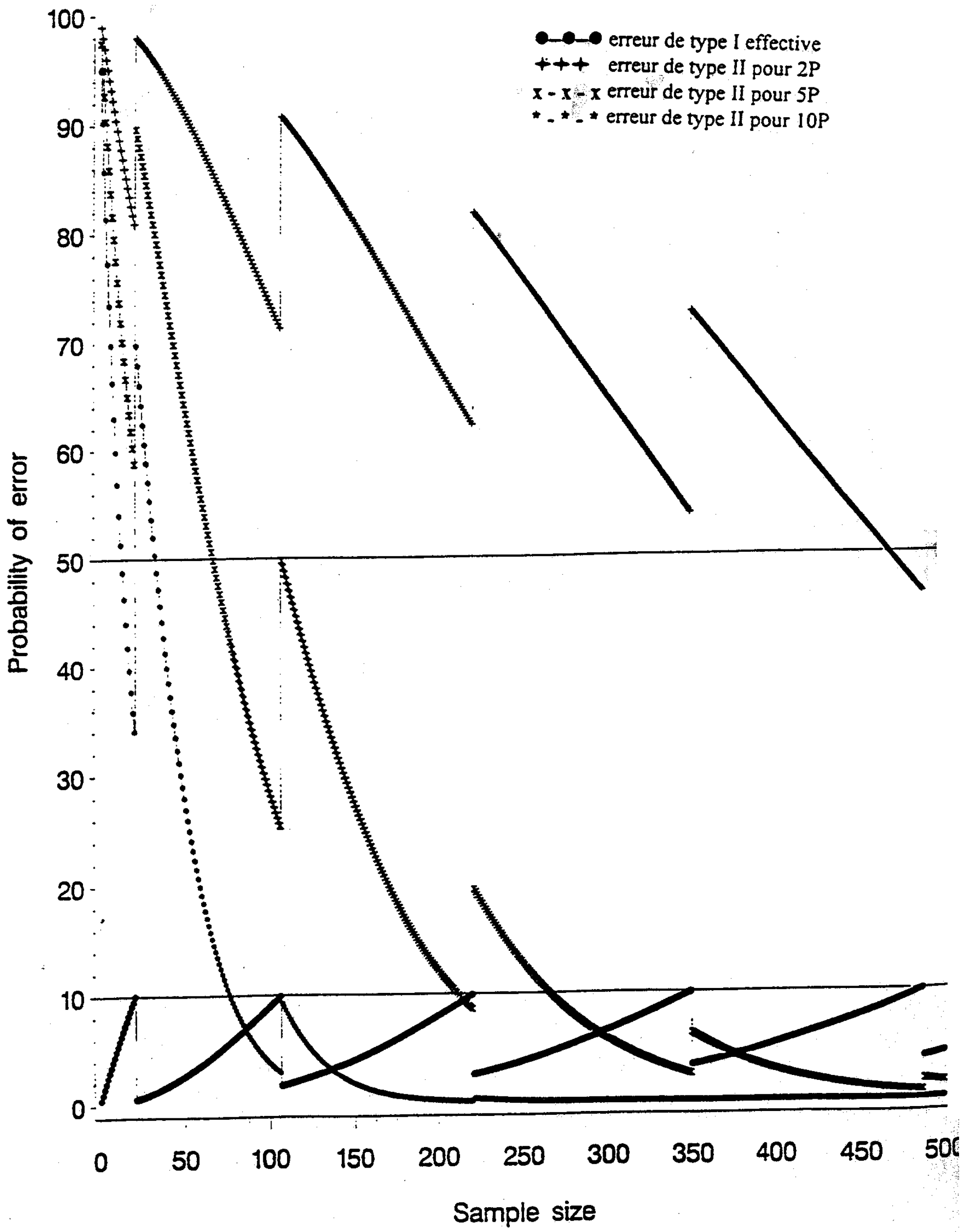


Table and figure 6: **Population Standard = .1%**
Acceptance Probability ≥90%
n=sample size, k=maximum number of off-types

n	k
1- 105	0
106- 532	1
533-1102	2
1103-1745	3
1746-2433	4
2434-3000	5

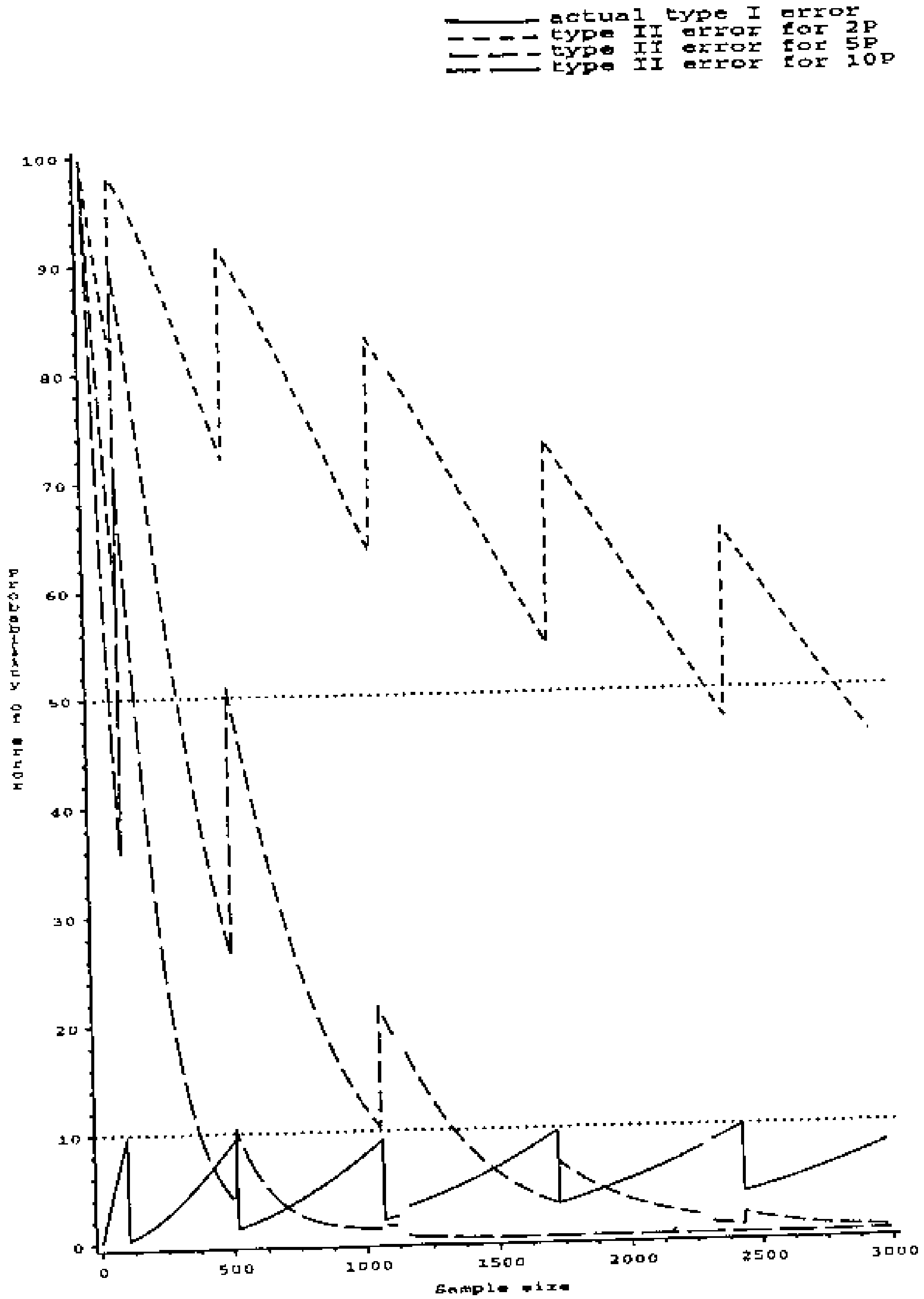


Table and figure 7: **Population Standard = 5%**
Acceptance Probability ≥95%
n=sample size, k=maximum number of off-types

n	k
1- 1	0
2- 7	1
8- 16	2
17- 28	3
29- 40	4
41- 53	5
54- 67	6
68- 81	7
82- 95	8
96- 110	9
111- 125	10
126- 140	11
141- 155	12
156- 171	13
172- 187	14
188- 203	15
204- 219	16
220- 235	17
236- 251	18
252- 268	19
269- 284	20
285- 300	21
301- 317	22
318- 334	23
335- 351	24
352- 367	25
368- 384	26
385- 401	27
402- 418	28
419- 435	29
436- 452	30
453- 469	31
470- 487	32
488- 504	33
505- 521	34
522- 538	35
539- 556	36
557- 573	37
574- 590	38
591- 608	39
609- 625	40
626- 643	41
644- 660	42
661- 678	43
679- 696	44
697- 713	45
714- 731	46
732- 748	47
749- 766	48
767- 784	49
785- 802	50
803- 819	51
820- 837	52
838- 855	53
856- 873	54
874- 891	55
892- 909	56
910- 926	57
927- 944	58
945- 962	59
963- 980	60
981- 998	61

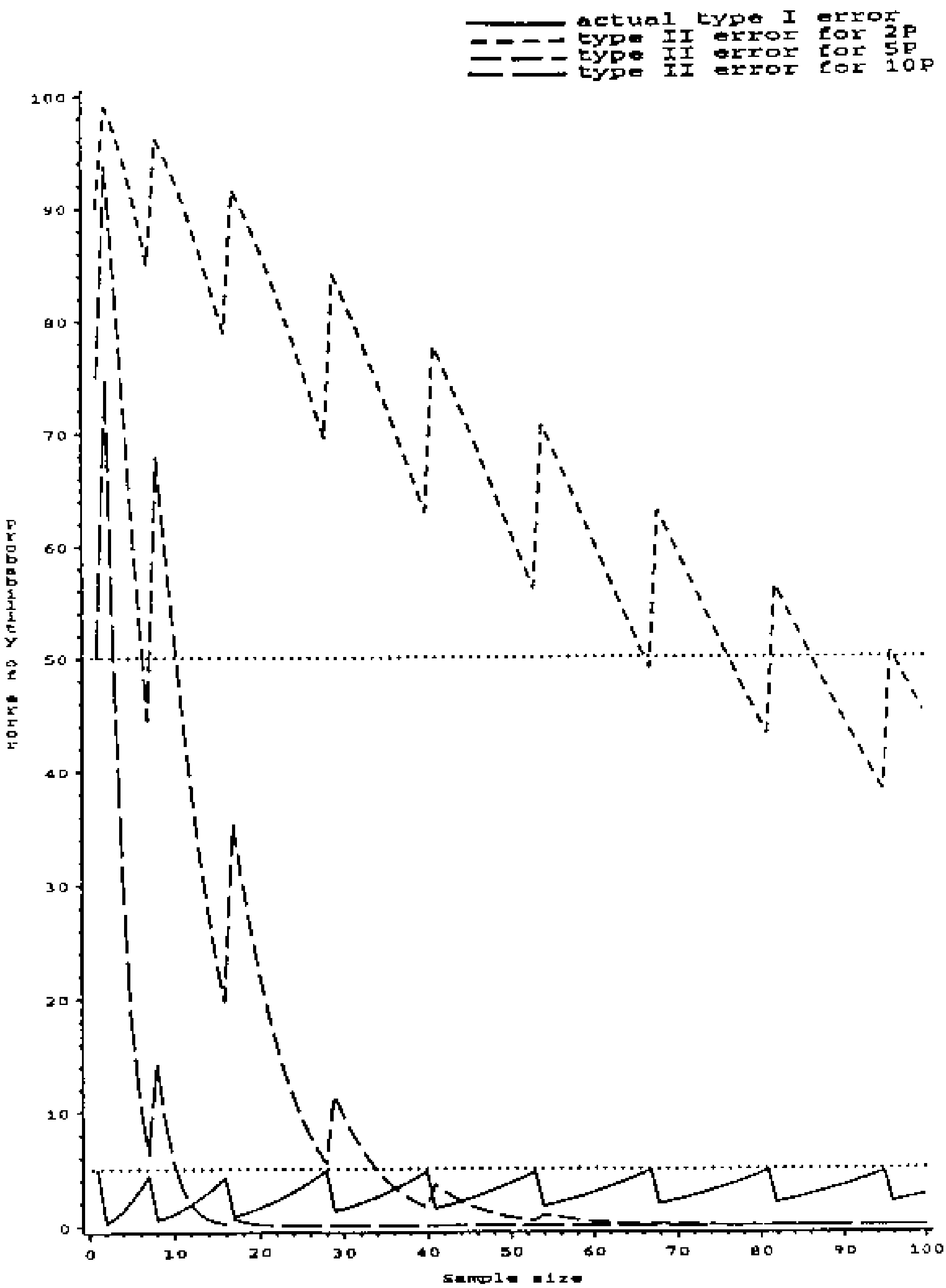


Table and figure 8: **Population Standard = 3%**
Acceptance Probability ≥95%
n=sample size, k=maximum number of off-types

n	k
1- 1	0
2- 12	1
13- 27	2
28- 46	3
47- 66	4
67- 88	5
89- 110	6
111- 134	7
135- 158	8
159- 182	9
183- 207	10
208- 232	11
233- 258	12
259- 284	13
285- 310	14
311- 337	15
338- 363	16
364- 390	17
391- 417	18
418- 444	19
445- 472	20
473- 499	21
500- 527	22
528- 554	23
555- 582	24
583- 610	25
611- 638	26
639- 666	27
667- 695	28
696- 723	29
724- 751	30
752- 780	31
781- 809	32
810- 837	33
838- 866	34
867- 895	35
896- 924	36
925- 952	37
953- 981	38
982-1010	39
1011-1040	40
1041-1069	41
1070-1098	42
1099-1127	43
1128-1156	44
1157-1186	45
1187-1215	46
1216-1244	47
1245-1274	48
1275-1303	49
1304-1333	50
1334-1362	51
1363-1392	52
1393-1422	53
1423-1451	54
1452-1481	55
1482-1511	56
1512-1541	57
1542-1570	58
1571-1600	59
1601-1630	60
1631-1660	61

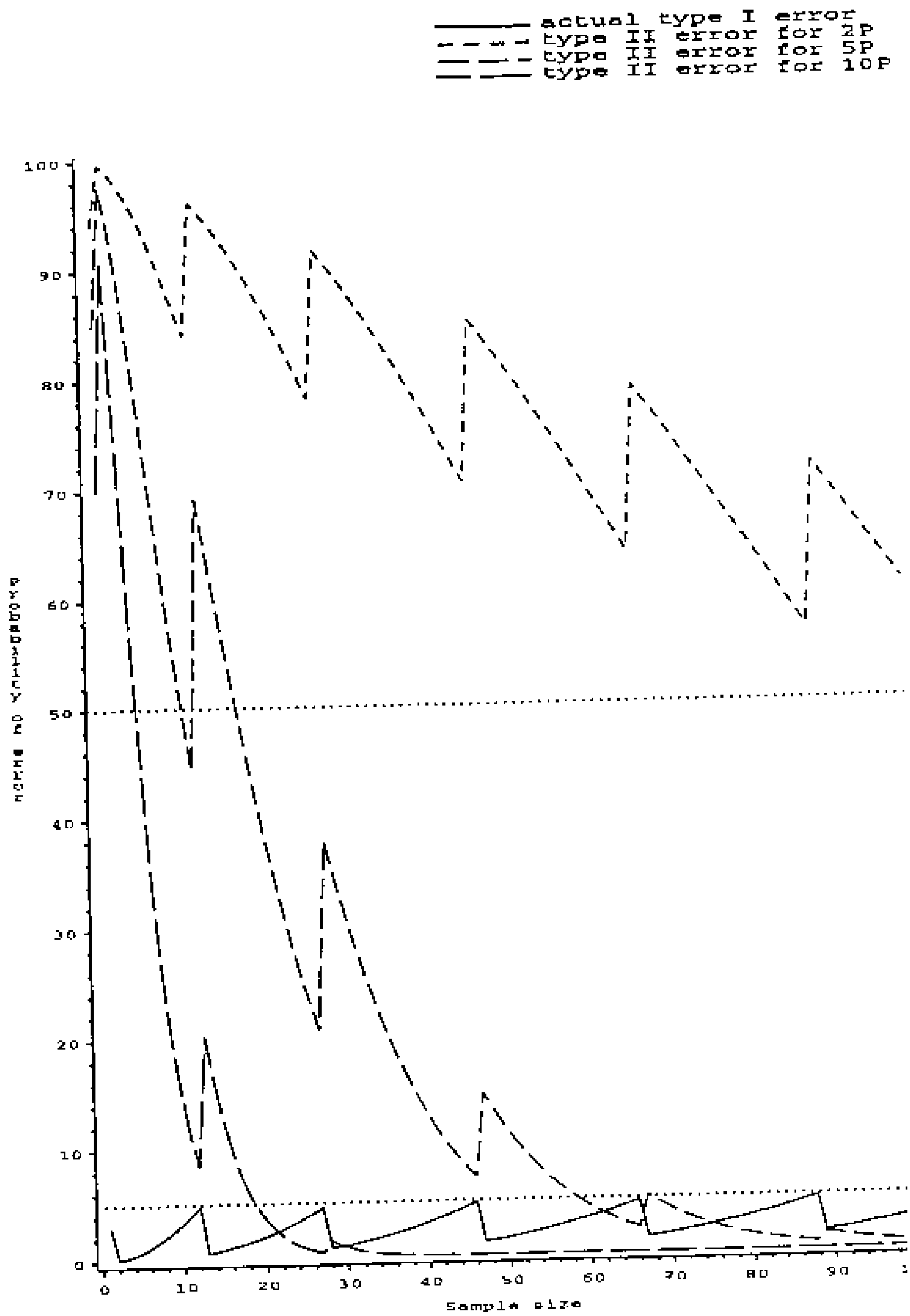


Table and figure 8 continued:

1423- 1451	54
1452- 1481	55
1482- 1511	56
1512- 1541	57
1542- 1570	58
1571- 1600	59
1601- 1630	60
1631- 1660	61
1661- 1690	62
1691- 1720	63
1721- 1750	64
1751- 1780	65
1781- 1810	66
1811- 1840	67
1841- 1870	68
1871- 1900	69
1901- 1930	70
1931- 1960	71
1961- 1990	72
1991- 2000	73

Table and figure 9: **Population Standard = 2%**
Acceptance Probability ≥95%
n=sample size, k=maximum number of off-types

n	k
1- 2	0
3- 18	1
19- 41	2
42- 69	3
70- 99	4
100- 131	5
132- 165	6
166- 200	7
201- 236	8
237- 273	9
274- 310	10
311- 348	11
349- 386	12
387- 425	13
426- 464	14
465- 504	15
505- 544	16
545- 584	17
585- 624	18
625- 665	19
666- 706	20
707- 747	21
748- 789	22
790- 830	23
831- 872	24
873- 914	25
915- 956	26
957- 998	27
999-1040	28
1041-1083	29
1084-1126	30
1127-1168	31
1169-1211	32
1212-1254	33
1255-1297	34
1298-1340	35
1341-1383	36
1384-1427	37
1428-1470	38
1471-1514	39
1515-1557	40
1558-1601	41
1602-1645	42
1646-1689	43
1690-1732	44
1733-1776	45
1777-1820	46
1821-1864	47
1865-1909	48
1910-1953	49
1954-1997	50
1998-2000	51

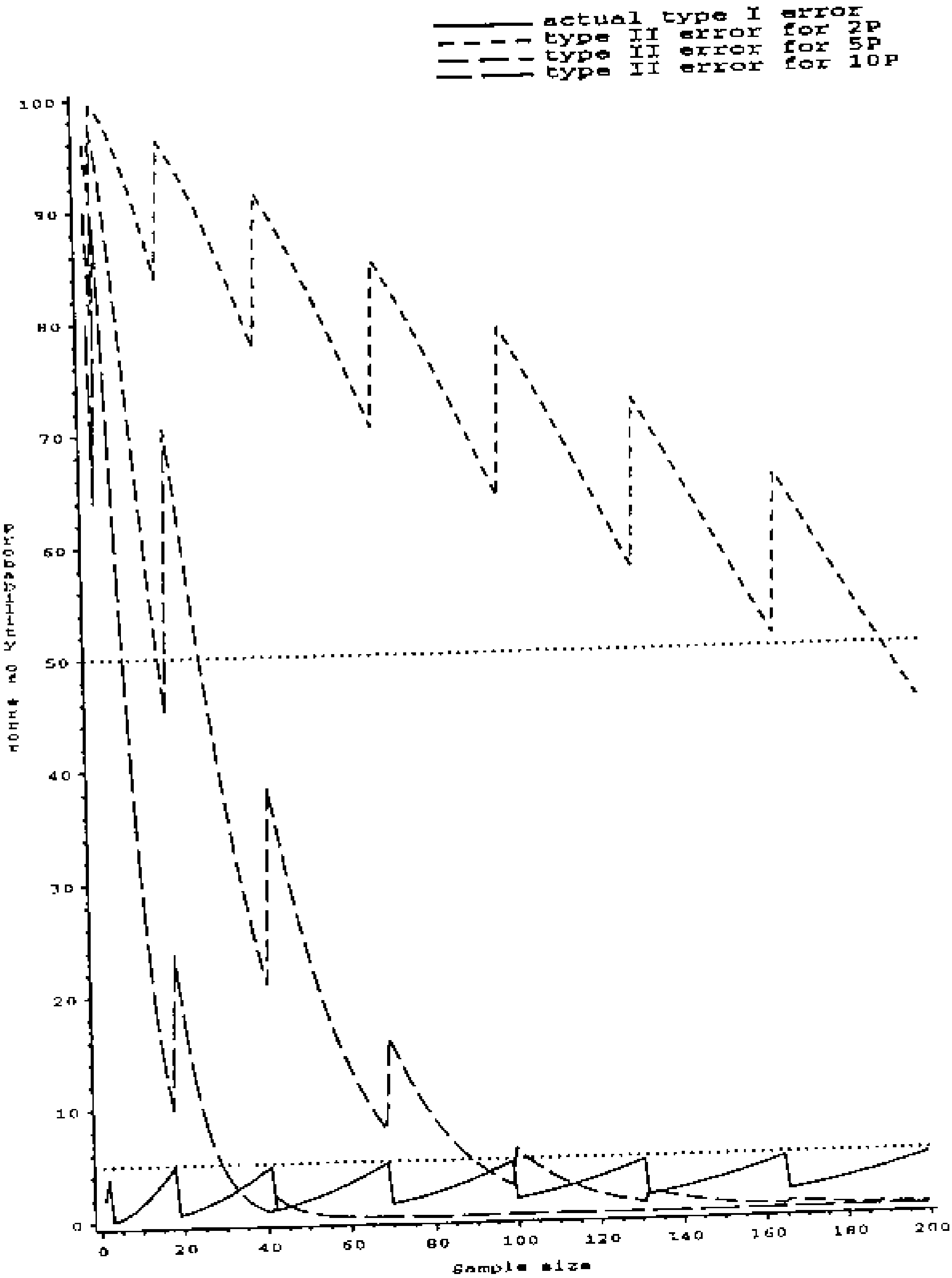


Table and figure 10: **Population Standard = 1%**
Acceptance Probability ≥95%
n=sample size, k=maximum number of off-types

n	k
1- 5	0
6- 35	1
36- 82	2
83- 137	3
138- 198	4
199- 262	5
263- 329	6
330- 399	7
400- 471	8
472- 544	9
545- 618	10
619- 694	11
695- 771	12
772- 848	13
849- 927	14
928-1006	15
1007-1085	16
1086-1166	17
1167-1246	18
1247-1328	19
1329-1410	20
1411-1492	21
1493-1575	22
1576-1658	23
1659-1741	24
1742-1825	25
1826-1909	26
1910-1993	27
1994-2078	28
2079-2163	29
2164-2248	30
2249-2333	31
2334-2419	32
2420-2505	33
2506-2593	34
2592-2677	35
2678-2763	36
2764-2850	37
2851-2937	38
2938-3000	39

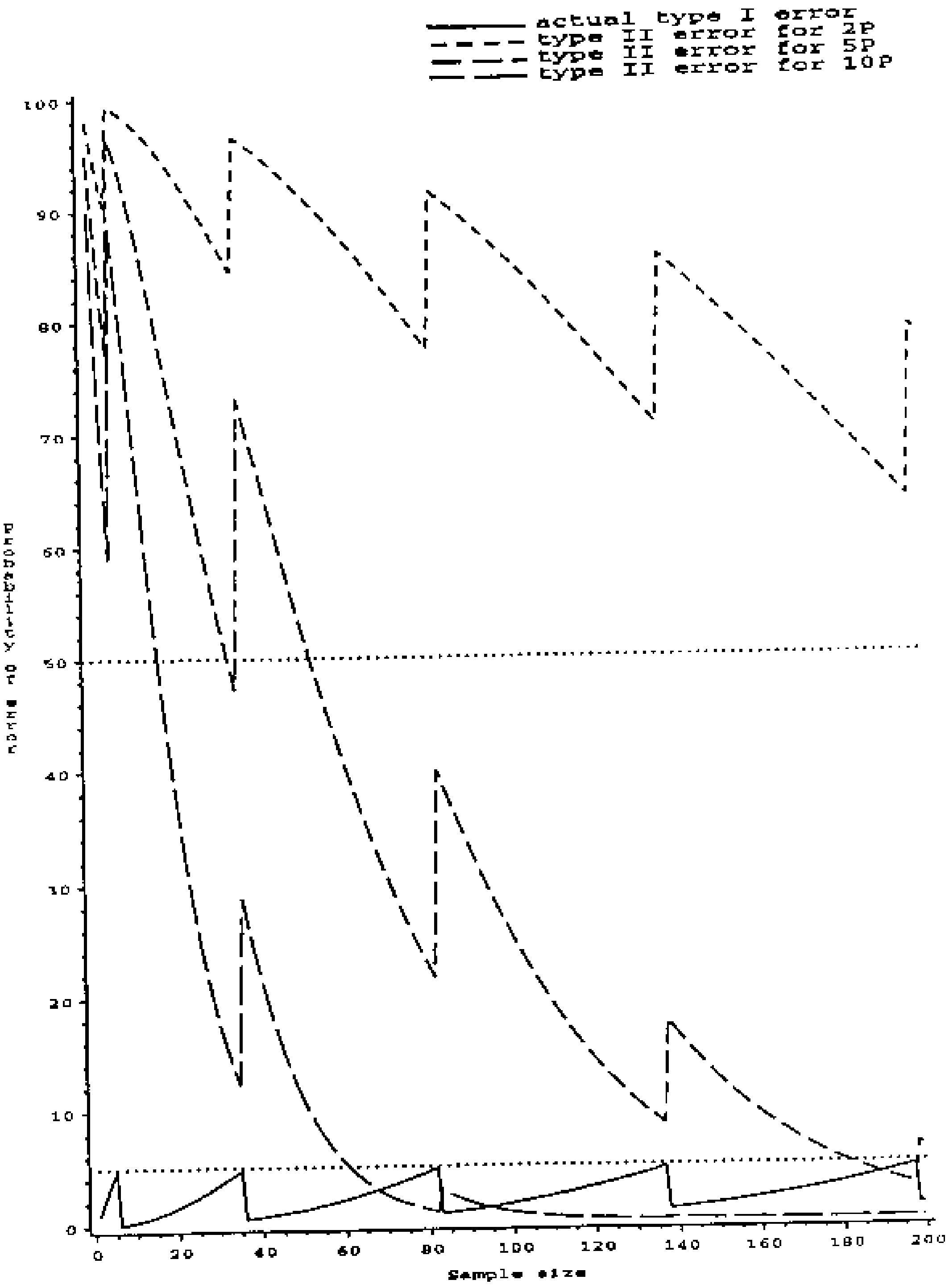


Table and figure 11: **Population Standard = .5%**
Acceptance Probability ≥95%
n=sample size, k=maximum number of off-types

n	k
1- 10	0
11- 71	1
72- 164	2
165- 274	3
275- 395	4
396- 523	5
524- 658	6
659- 797	7
798- 940	8
941-1086	9
1087-1235	10
1236-1386	11
1387-1540	12
1541-1695	13
1696-1851	14
1852-2009	15
2010-2169	16
2170-2329	17
2330-2491	18
2492-2653	19
2654-2817	20
2818-2981	21
2982-3000	22

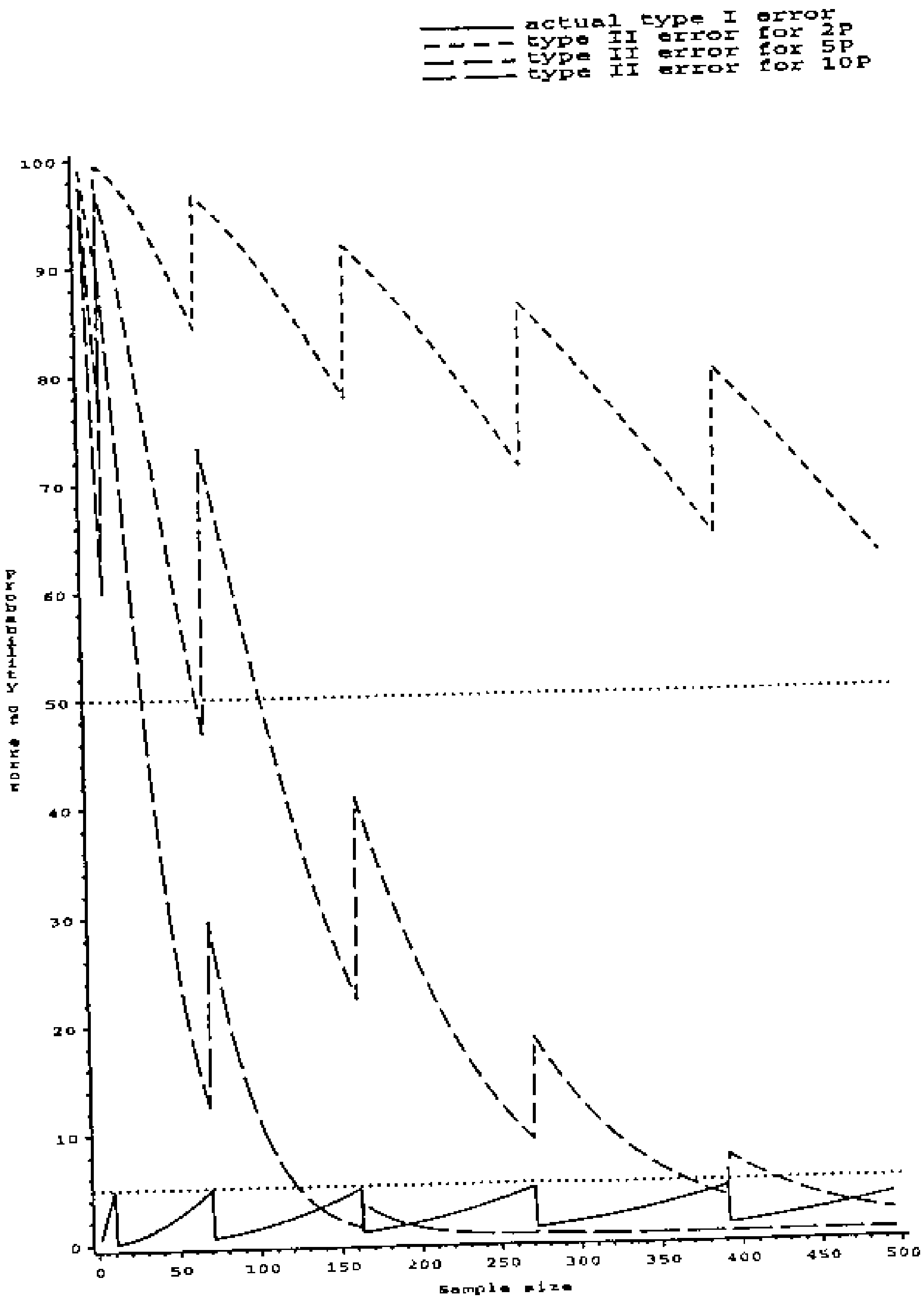


Table and figure 12: **Population Standard = .1%**
Acceptance Probability ≥95%
n=sample size, k=maximum number off-types

1- 51	0
52- 355	1
356- 818	2
819-1367	3
1368-1971	4
1972-2614	5
2615-3000	6

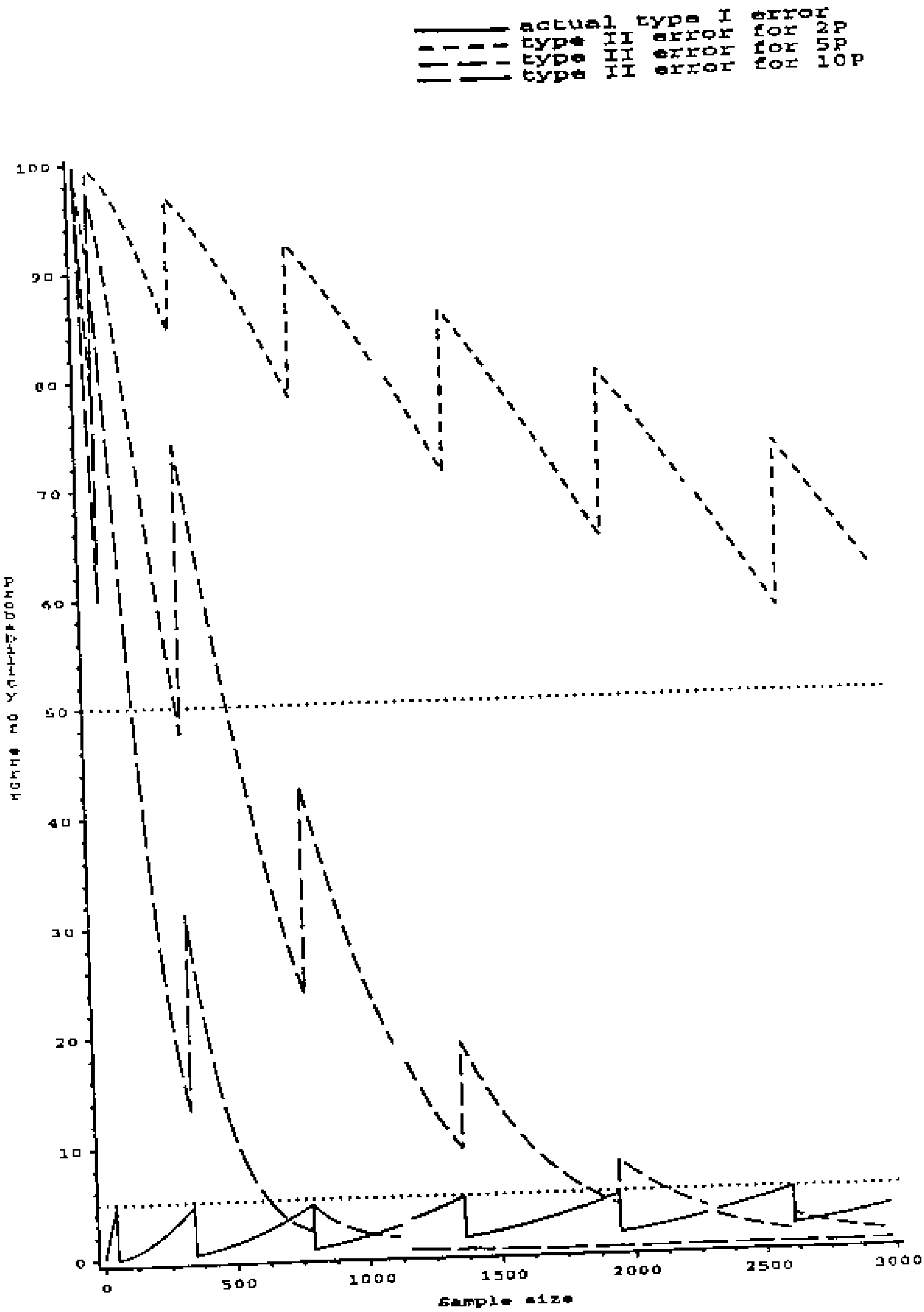


Table and figure 13: **Population Standard = 5%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

1- 3	1
4- 9	2
10- 17	3
18- 26	4
27- 37	5
38- 48	6
49- 60	7
61- 72	8
73- 85	9
86- 98	10
99- 111	11
112- 124	12
125- 138	13
139- 152	14
153- 167	15
168- 181	16
182- 196	17
197- 210	18
211- 225	19
226- 240	20
241- 255	21
256- 270	22
271- 286	23
287- 301	24
302- 317	25
318- 332	26
333- 348	27
349- 364	28
365- 380	29
381- 395	30
396- 411	31
412- 427	32
428- 444	33
445- 460	34
461- 476	35
477- 492	36
493- 508	37
509- 525	38
526- 541	39
542- 558	40
559- 574	41
575- 591	42
592- 607	43
608- 624	44
625- 640	45
641- 657	46
658- 674	47
675- 690	48
691- 707	49
708- 724	50
725- 741	51
742- 758	52
759- 775	53
776- 792	54
793- 809	55
810- 826	56

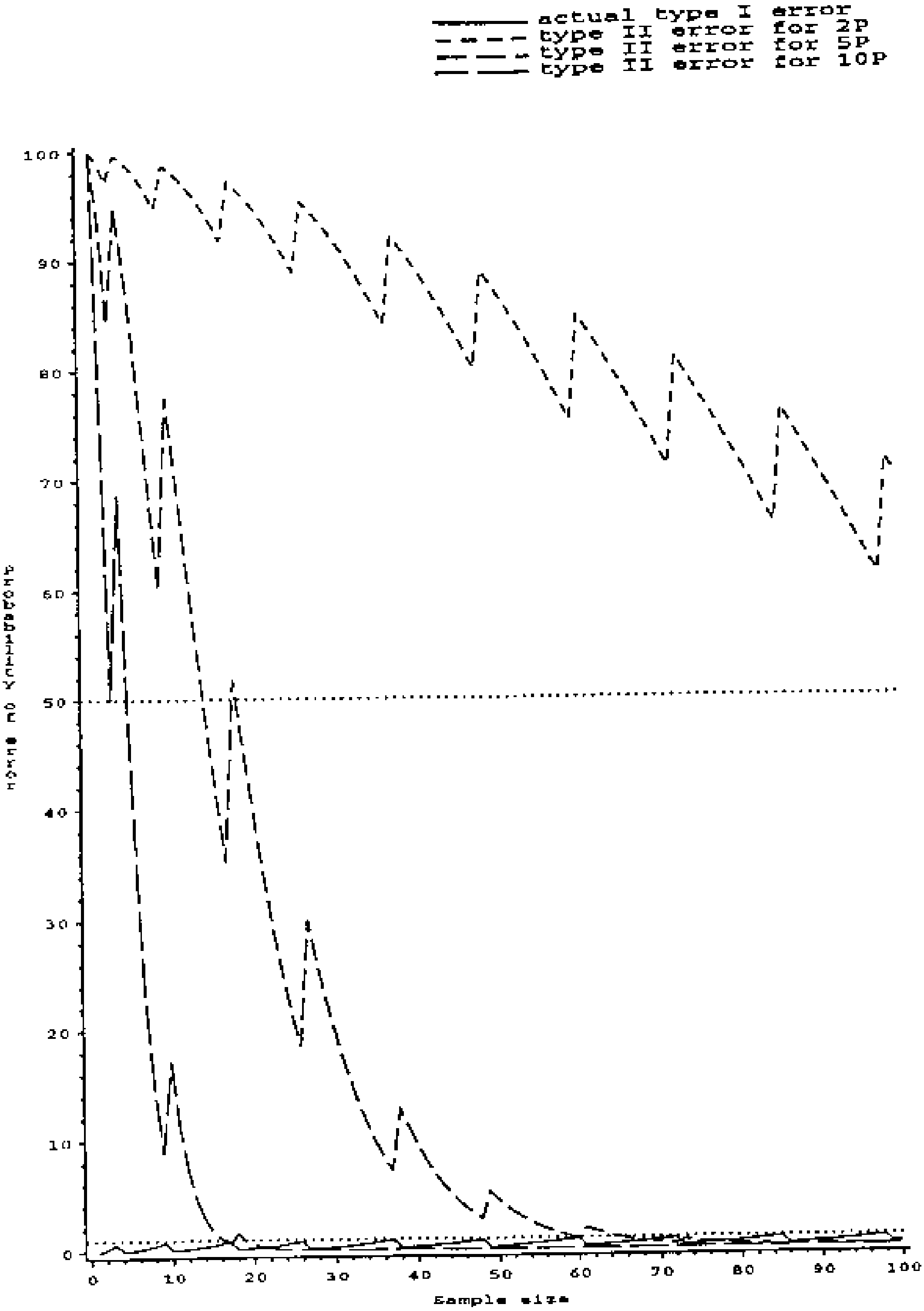


Table and figure 13 continued:

793-	809	55
810-	826	56
827-	843	57
844-	860	58
861-	877	59
878-	894	60
895-	911	61
912-	928	62
929-	945	63
946-	962	64
963-	979	65
980-	997	66
998-	1014	67
1015-	1031	68
1032-	1048	69
1049-	1066	70
1067-	1083	71
1084-	1100	72
1101-	1118	73
1119-	1135	74
1136-	1153	75
1154-	1170	76
1171-	1187	77
1188-	1205	78
1206-	1222	79
1223-	1240	80
1241-	1257	81
1258-	1275	82
1276-	1292	83
1293-	1310	84
1311-	1327	85
1328-	1345	86
1346-	1362	87
1363-	1380	88
1381-	1398	89
1399-	1415	90
1416-	1433	91
1434-	1451	92
1452-	1468	93
1469-	1486	94
1487-	1504	95
1505-	1521	96
1522-	1539	97
1540-	1557	98
1558-	1574	99
1575-	1592	100
1593-	1610	101
1611-	1628	102
1629-	1645	103
1646-	1663	104
1664-	1681	105
1682-	1699	106
1700-	1717	107
1718-	1734	108
1735-	1752	109
1753-	1770	110
1771-	1788	111
1789-	1806	112

Table and figure 14: **Population Standard = 3%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

n	k
1- 5	1
6- 15	2
16- 28	3
29- 44	4
45- 61	5
62- 79	6
80- 98	7
99- 119	8
120- 140	9
141- 161	10
162- 183	11
184- 206	12
207- 229	13
230- 252	14
253- 276	15
277- 300	16
301- 324	17
325- 348	18
349- 373	19
374- 398	20
399- 423	21
424- 448	22
449- 474	23
475- 499	24
500- 525	25
526- 551	26
552- 577	27
578- 603	28
604- 629	29
630- 656	30
657- 682	31
683- 709	32
710- 736	33
737- 763	34
764- 789	35
790- 816	36
817- 844	37
845- 871	38
872- 898	39
899- 925	40
926- 953	41
954- 980	42
981-1008	43
1009-1035	44
1036-1063	45
1064-1091	46
1092-1119	47
1120-1146	48
1147-1174	49
1175-1202	50
1203-1230	51
1231-1258	52
1259-1286	53
1287-1315	54
1316-1343	55
1344-1371	56
1372-1399	57
1400-1428	58
1429-1456	59
1457-1484	60
1485-1513	61

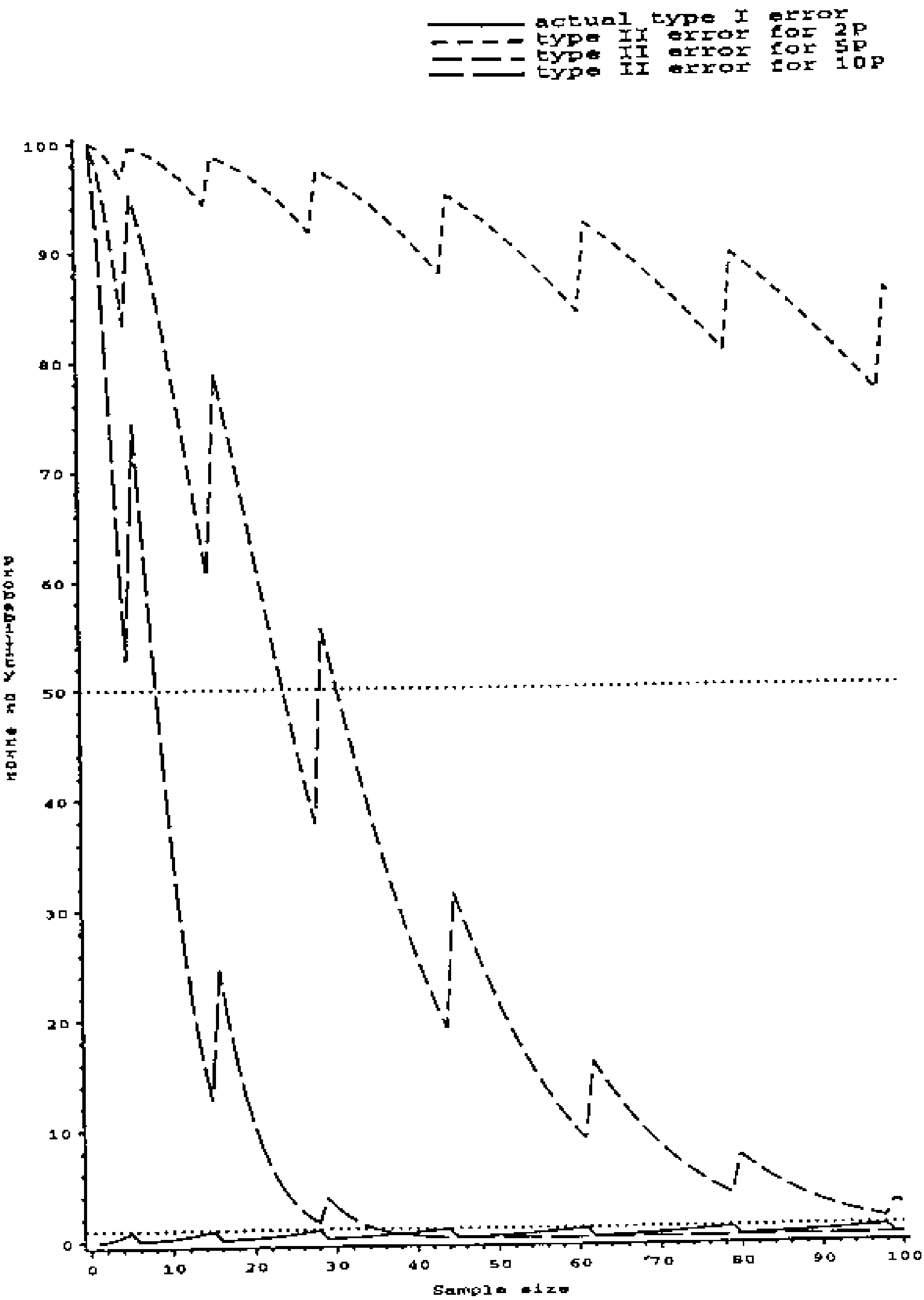


Table and figure 15: **Population Standard = 2%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

n	k
1- 7	1
8- 22	2
23- 42	3
43- 65	4
66- 90	5
91- 118	6
119- 147	7
148- 177	8
178- 208	9
209- 241	10
242- 274	11
275- 307	12
308- 342	13
343- 377	14
378- 412	15
413- 448	16
449- 484	17
485- 521	18
522 558	19
559- 595	20
596- 632	21
633- 670	22
671- 708	23
709- 747	24
748- 785	25
786- 824	26
825- 863	27
864- 902	28
903- 942	29
943- 981	30
982-1021	31
1022-1061	32
1062-1101	33
1102-1141	34
1142-1182	35
1183-1222	36
1223-1263	37
1264-1303	38
1304-1344	39
1345-1385	40
1386-1426	41
1427-1467	42
1468-1509	43
1510-1550	44
1551-1591	45
1592-1633	46
1634-1675	47
1676-1716	48
1717-1758	49
1759-1800	50
1801-1842	51
1843-1884	52
1885-1926	53
1927-1968	54
1969-2000	55

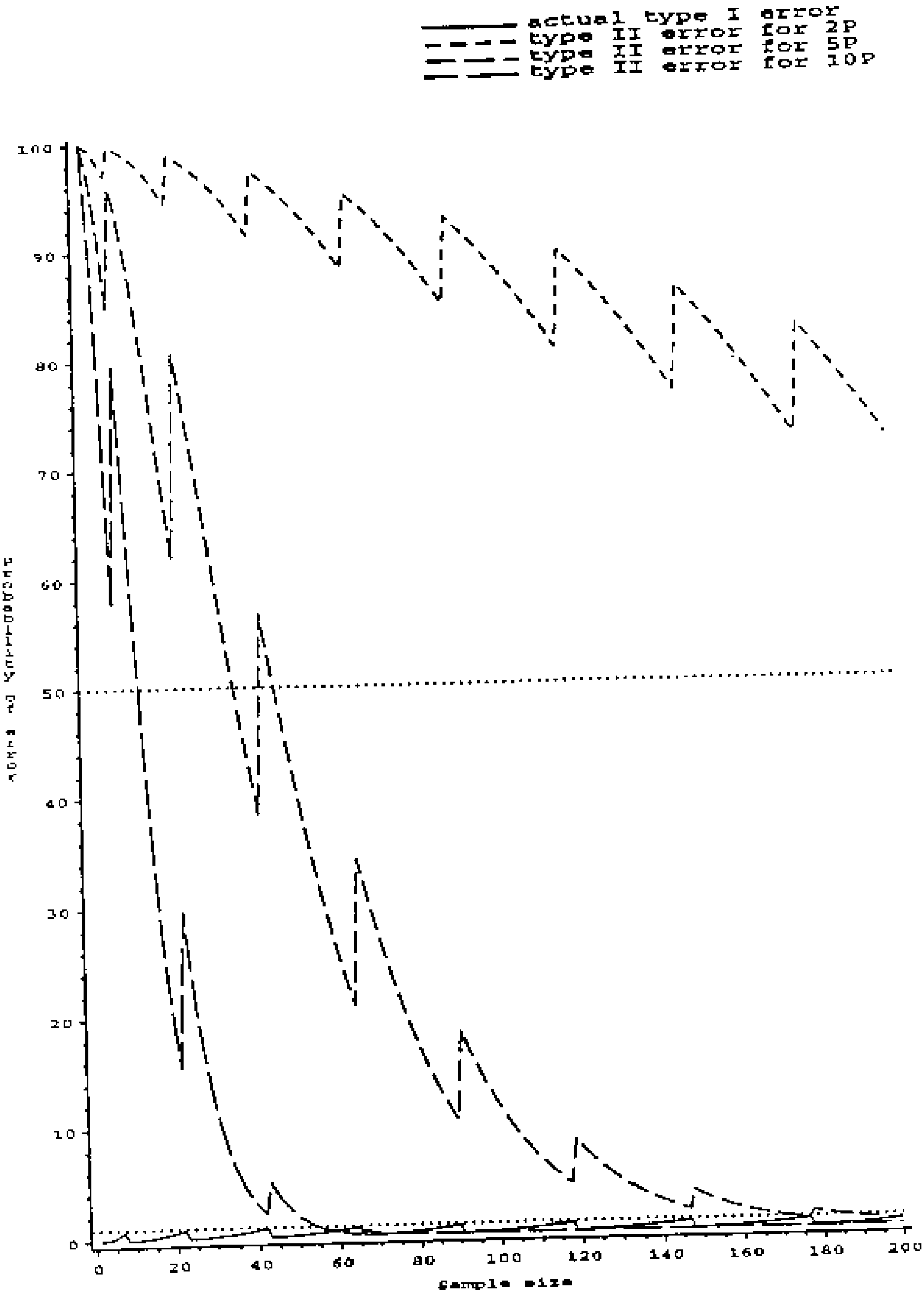


Table and figure 16: **Population Standard = 1%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

n	k
1- 1	0
2- 15	1
16- 44	2
45- 83	3
84- 129	4
130- 180	5
181- 234	6
235- 292	7
293- 353	8
354- 415	9
416- 479	10
480- 545	11
546- 612	12
613- 681	13
682- 750	14
751- 821	15
822- 893	16
894- 965	17
966-1038	18
1039-1112	19
1113-1186	20
1187-1261	21
1262-1337	22
1338-1413	23
1414-1489	24
1490-1566	25
1567-1644	26
1645-1722	27
1723-1800	28
1801-1879	29
1880-1958	30
1959-2037	31
2038-2117	32
2118-2197	33
2198-2277	34
2278-2358	35
2359-2439	36
2440-2520	37
2521-2601	38
2602-2683	39
2684-2764	40
2765-2846	41
2847-2929	42
2930-3000	43

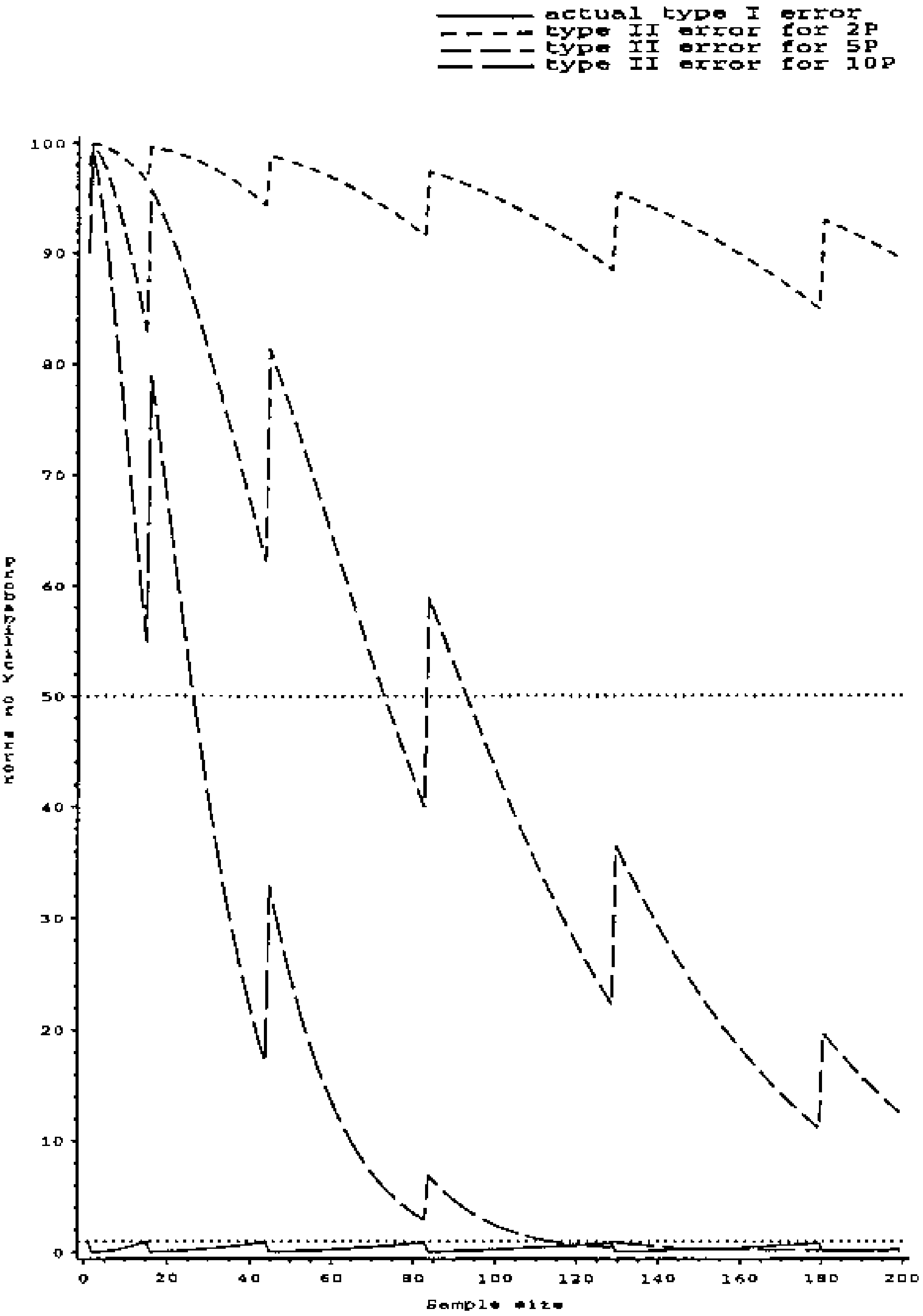


Table and figure 17: **Population Standard = .5%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

n	k
1- 2	0
3- 30	1
31- 87	2
88- 165	3
166- 257	4
258- 358	5
359- 467	6
468- 583	7
584- 703	8
704- 828	9
829- 956	10
957-1088	11
1089-1222	12
1223-1359	13
1360-1498	14
1499-1639	15
1640-1782	16
1783-1926	17
1927-2072	18
2073-2220	19
2221-2369	20
2370-2519	21
2520-2670	22
2671-2822	23
2823-2975	24
2976-3000	25

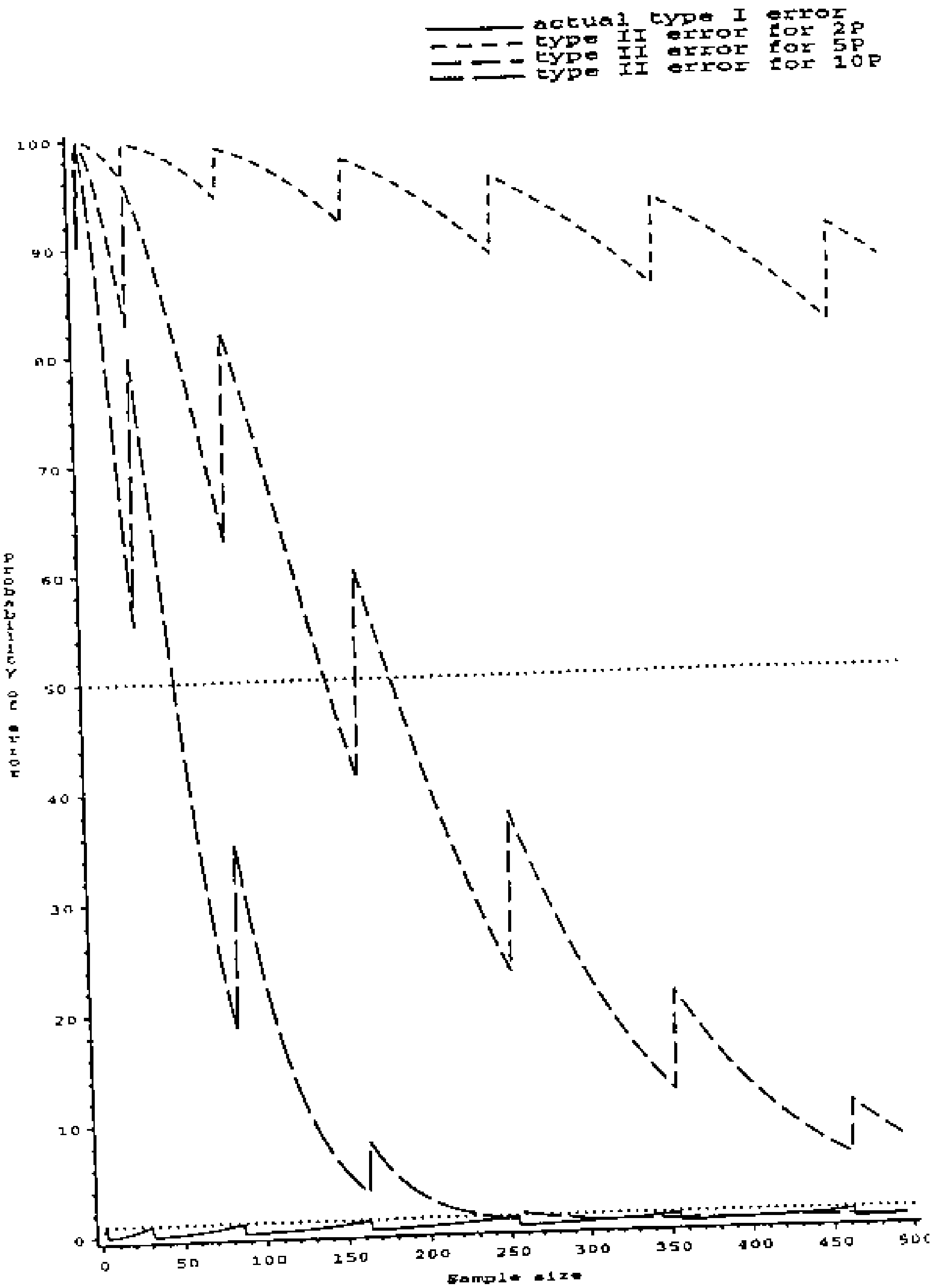


Table and figure 18: **Population Standard = .1%**
Acceptance Probability ≥99%
n=sample size, k=maximum number of off-types

n	k
1- 10	0
11- 148	1
149- 436	2
437- 824	3
825-1280	4
1281-1786	5
1787-2332	6
2333-2908	7
2909-3000	8

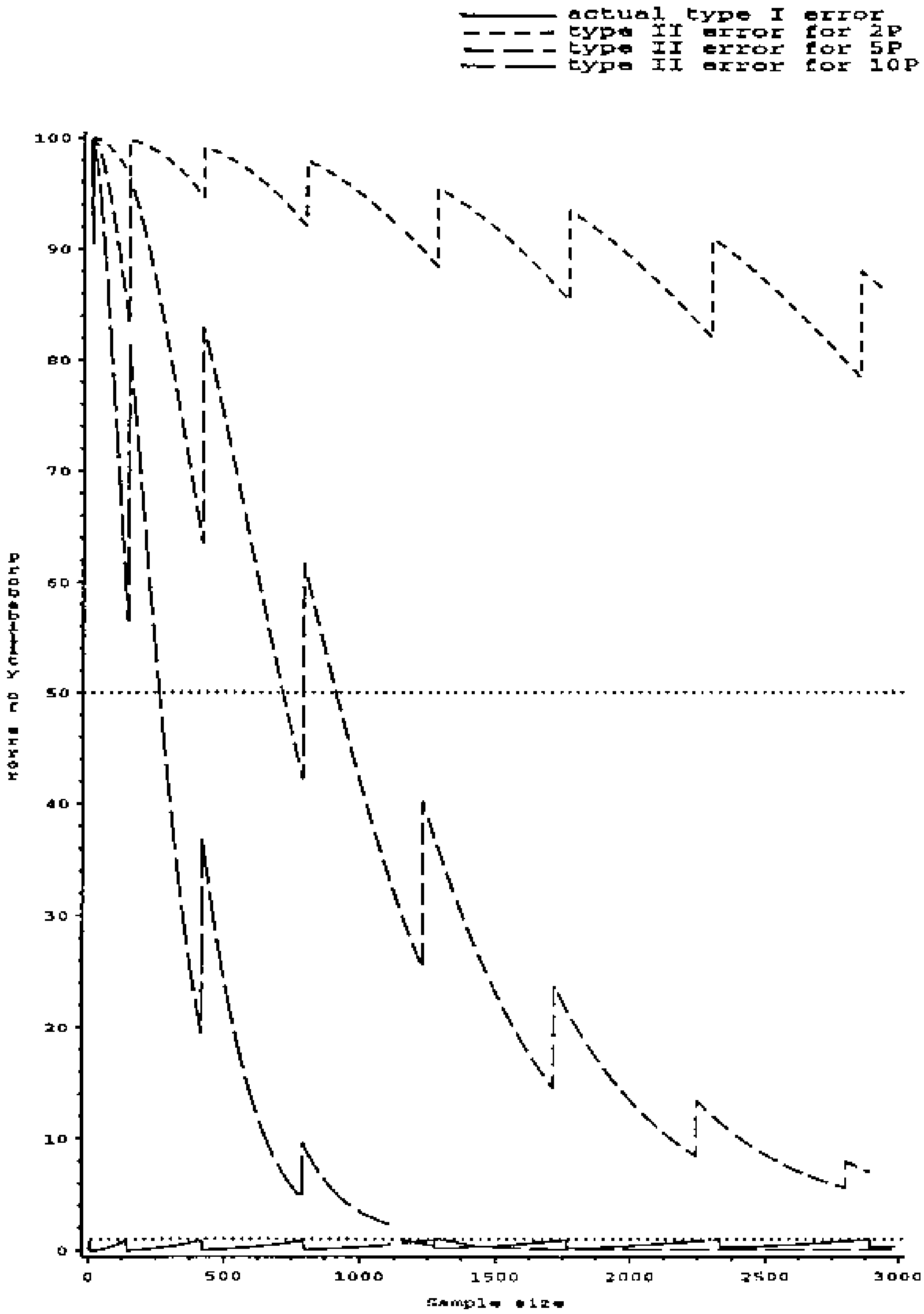


Table and figure 19: **Population Standard = 10%**
Acceptance Probability ≥ 90%
n=sample size, k=maximum number of off-types

1-	1	0
2-	5	1
6-	11	2
12-	18	3
19-	25	4
26-	32	5
33-	40	6
41-	47	7
48	55	8
56-	63	9
64-	71	10
72-	79	11
80-	88	12
89-	96	13
97-	104	14
105-	113	15
114-	121	16
122-	130	17
131-	138	18
139-	147	19
148-	156	20
157	164	21
165	173	22
174-	182	23
183-	191	24
192-	199	25
200-	200	26

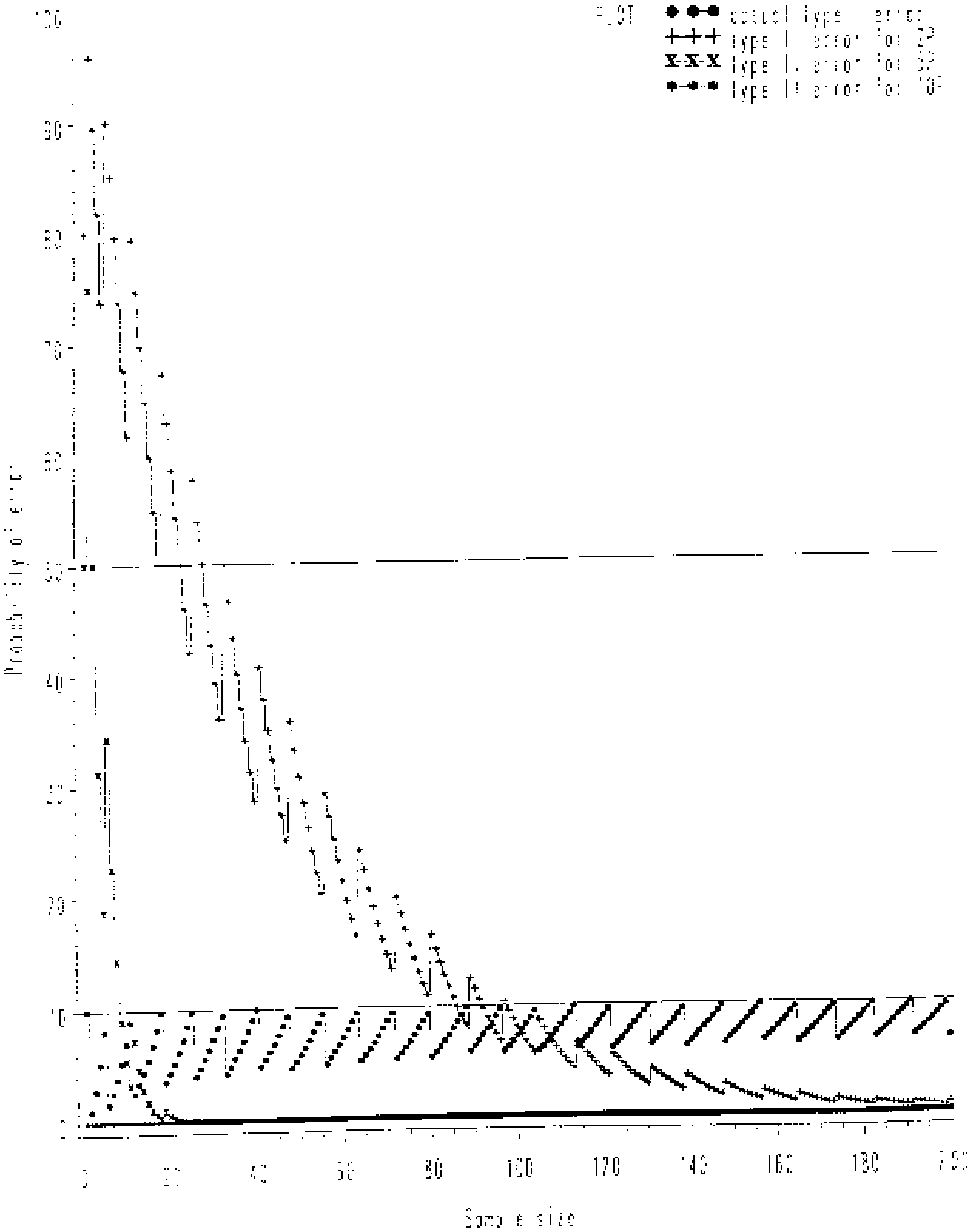


Table and figure 20: **Population Standard = 10%**
Acceptance Probability ≥ 95%
n=sample size, k=maximum number of off-types

n		k
1-	3	1
4-	8	2
9-	14	3
15-	20	4
21-	27	5
28-	34	6
35-	41	7
42-	48	8
49-	56	9
57-	63	10
64-	71	11
72-	79	12
80-	86	13
87	94	14
95-	102	15
103-	110	16
111-	119	17
120-	127	18
128-	135	19
136	143	20
144-	152	21
153-	160	22
161-	168	23
169-	177	24
178-	185	25
186-	194	26
195-	200	27

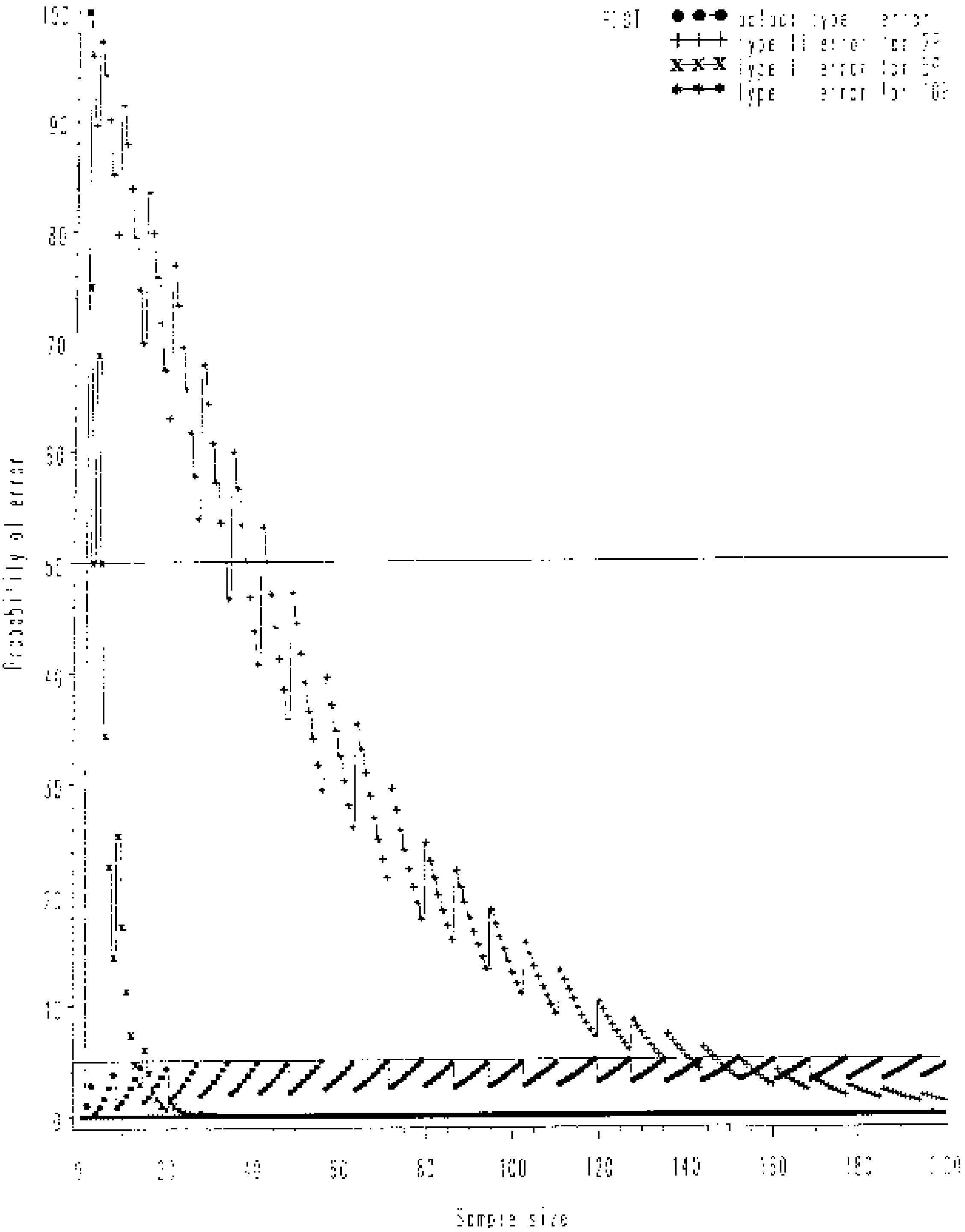
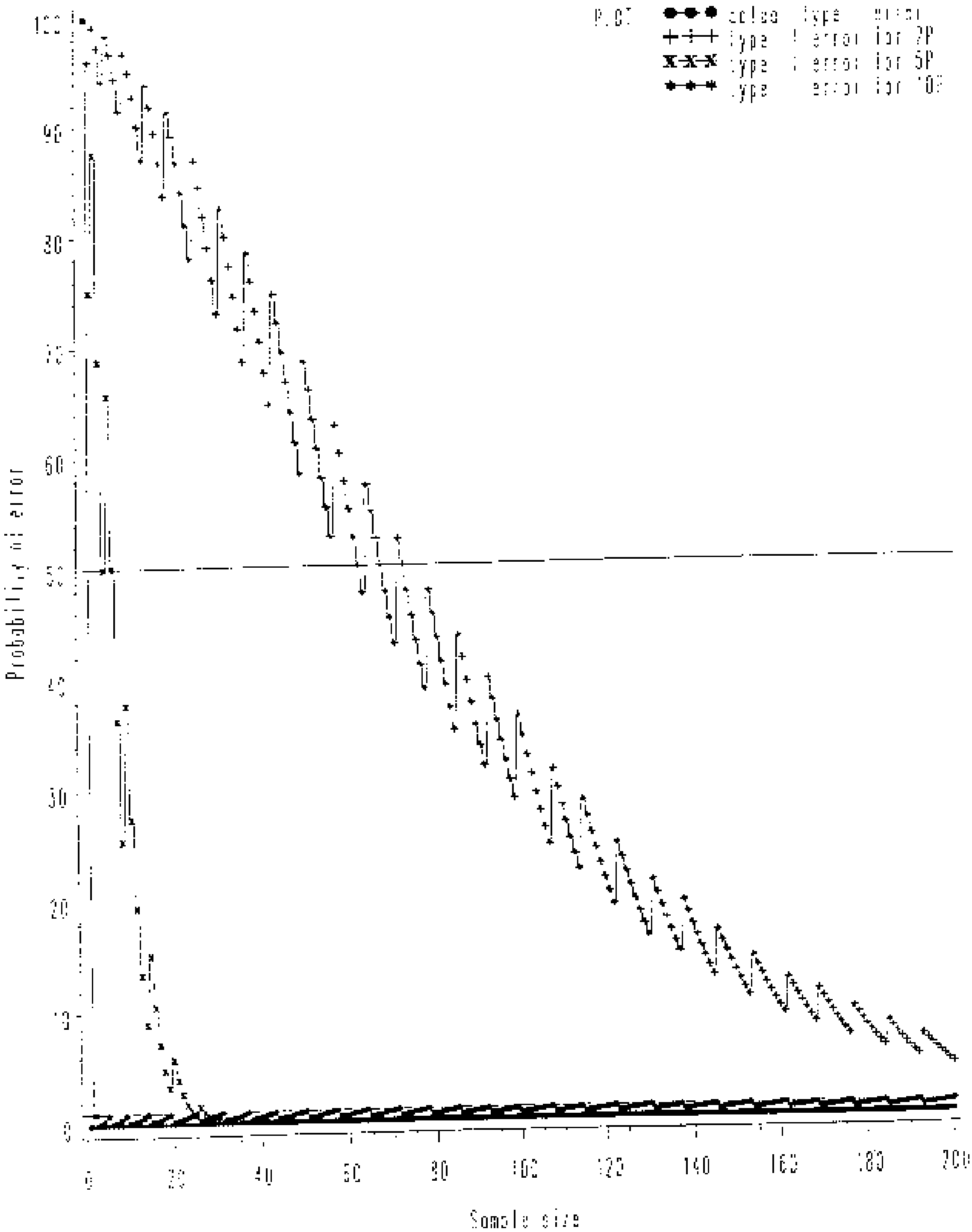


Table and figure 21 : **Population Standard = 10%**
Acceptance Probability ≥ 99%
n=sample size, k=maximum number of off-types

--		
1-	2	1
3-	5	2
6-	9	3
10-	14	4
15-	19	5
20-	25	6
26-	31	7
32-	37	8
38-	43	9
44-	50	10
51-	57	11
58-	64	12
65-	71	13
72-	78	14
79-	85	15
86-	92	16
93-	99	17
100-	107	18
108-	114	19
115-	122	20
123-	130	21
131-	137	22
138-	145	23
146-	153	24
154-	161	25
162-	168	26
169-	176	27
177-	184	28
185-	192	29
193-	200	30



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